This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

THE MEAN DENSITY OF THE EARTH.

THE

MEAN DENSITY OF THE EARTH.

AN ESSAY

TO WHICH THE ADAMS PRIZE WAS ADJUDGED IN 1893
IN THE UNIVERSITY OF CAMBRIDGE.

BY

J. H. POYNTING, Sc.D., F.R.S.,
LATE FELLOW OF TRINITY COLLEGE, CAMBRIDGE; PROFESSOR OF PHYSICS,
MASON COLLEGE, BIRMINGHAM.

> "Fortune him hath enhaunced so in pryde,
> That verraily he wend he might atteygne
> Unto the sterris upon every syde;
> And in a balance weyen ech mounteyne."

WITH ILLUSTRATIONS AND SEVEN FOLDING PLATES.

LONDON:
CHARLES GRIFFIN & COMPANY, LIMITED,
EXETER STREET, STRAND.
1894.

PREFACE.

THE subject for the Adams Prize for 1893 was "The Methods of determining the absolute and relative value of gravitation and the mean density of the earth."

This Essay, to which the Prize was adjudged, gives an account of the various experiments, including one by the author, which have been made to determine the mean density of the earth.

The First Part consists of an introductory discussion of the problem and an account of the experiments hitherto made on the subject. Since the Essay was first written I have had access to some of the original papers, of which I had before only seen abstracts or brief notices, and I have therefore been able to make the history of the subject more complete.

The Second Part contains an account of my own experiment which has already appeared in the *Philosophical Transactions of the Royal Society for* 1891. As it is a condition of the award of the Prize that the Essay should be printed, it is necessary to give here the account of the experiment in full as already published.

I have prefixed a bibliography of works and papers containing important or noteworthy contributions to the subject. In this will be found a reference to an experiment by M. Berget, of which the account has been published so recently that it could not be included in the First Part of the Essay.

CONTENTS.

PART I. HISTORICAL.

HISTORICAL ACCOUNT—

PLUMB-LINE EXPERIMENTS—

PENDULUM EXPERIMENTS—

TORSION BALANCE EXPERIMENTS—

PART II.

BIBLIOGRAPHY.

In this list I have only included papers which contain original matter, or which are of interest in the history of the subject. The titles of many others appearing to contain references to the mean density of the earth will be found in the Bibliography of the Pendulum, in vol. iv. "Mémoires Rélatifs à la Physique."

I have not seen any of these, but from the fact that they are not referred to in the papers describing original work, it is probable that they are unimportant, and I have omitted them.

1\. 1687. **NEWTON (SIR ISAAC).** PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA. Lib. 3. Phaenomena, and Propositiones I.–X.

Newton here shows that the observed phenomena agree with the supposition of universal gravitation according to the law GM_1M_2/d^2, and he deduces the relative masses and specific gravities of the sun and planets.

In Prop. X. he concludes that the earth may be five or six times as dense as water.

In Prop. VI. he describes the box pendulum experiments in which equal weights of different substances are in turn placed in a box forming the bob of a pendulum. The time of swing being the same in all cases, terrestrial gravitation is proportional to the mass and not selective.

2\. **NEWTON (SIR ISAAC).** DE MUNDI SYSTEMATE. (Opera, edited by Horsley, vol. iii. p. 194.)

Translated in vol. iii. of Davis's edition of Motte's version of the "Principia," 1803, p. 21.

In a passage on the attraction of terrestrial masses, Newton calculates the attraction of two spheres 1ft. in diameter, and of the same density as the earth, though there is a curious error in the calculation. He also estimates the plumb-line deflection of a hemispherical mountain, three miles high.

3\. 1749. **BOUGUER.** LA FIGURE DE LA TERRE. 4to. Paris, 1749. (Referred to in Todhunter's "Theory of Attractions."

Vol. i. pp. 247–8. Clarke's "Geodesy," pp. 324–6, and Maskelyne's paper No. 4 below.)

The 7th section gives an account of Bouguer's attempts to determine the plumb-line deflection due to Chimborazo, and the increase of gravity at Quito, due to the table-land on which it is situated.

The first work on the subject, though the results were recognised as quantitatively valueless by the experimenter.

4. 1772. **MASKELYNE (REV. NEVIL, B.D., F.R.S., and Astronomer Royal).** A PROPOSAL FOR MEASURING THE ATTRACTION OF SOME HILL IN THIS KINGDOM BY ASTRONOMICAL OBSERVATIONS. (*Philosophical Transactions*, 1775, p. 495.)

A proposal to use some mountain in England, either Whernside or Helvellyn and Skiddaw.

The reading of this paper resulted in the appointment of a committee, of which Maskelyne was a member, to find a suitable hill, and to carry out the necessary work. The surplus remaining from the grant made by George III. to defray the expenses of the observations of the 1769 Transit of Venus was given to the committee by the King.

5. 1775. **MASKELYNE (REV. N., B.D., F.R.S., and Astronomer Royal).** AN ACCOUNT OF OBSERVATIONS MADE ON THE MOUNTAIN SCHEHALLIEN FOR FINDING ITS ATTRACTION. (*Philosophical Transactions*, 1775, p. 500.)

This paper gives a full account of the astronomical work, carried out by Maskelyne, in this celebrated experiment. The zenith distances of a number of stars were observed at two stations respectively on the north and south sides of the mountain. There is also an account of the survey of the district.

The work of computation was undertaken by Hutton, and described by him in the paper No. 6 below.

6. 1778. **HUTTON (CHARLES, F.R.S.).** AN ACCOUNT OF THE CALCULATIONS MADE FROM THE SURVEY AND MEASURES TAKEN AT SCHEHALLIEN, IN ORDER TO ASCERTAIN THE MEAN DENSITY OF THE EARTH. (*Philosophical Transactions*, 1778, pp. 689–788. Revised in the Author's *Tracts*, vol. ii. p. 64, 1812.)

A very full account of the method of determining the difference of attractions at the north and south stations on Schehallien, in terms of the masses in the district. Value obtained for the mean density 4·5. In the revision of 1812 he gives 4·95.

See also Nos. 9 and 12 below.

7. 1780. **HUTTON (CHARLES, F.R.S.).** CALCULATIONS TO DETERMINE AT WHAT POINT IN THE SIDE OF A HILL ITS ATTRACTION WILL BE THE GREATEST, ETC. (*Philosophical Transactions*, 1780, p. 1.)

The maximum effect at about ¼ the altitude.

8. 1798. **CAVENDISH (H., F.R.S.).** EXPERIMENTS to DETERMINE THE DENSITY OF THE EARTH. (*Philosophical Transactions*, vol. lxxxviii., 1798, pp. 469–526.)

An account of the celebrated experiment devised by Michell, and carried out by Cavendish, to determine the attraction between lead spheres by means of the torsion balance. Value of the mean density 5.448.

9. 1811. **PLAYFAIR. (J.)** ACCOUNT OF A LITHOLOGICAL SURVEY OF SCHEHALLIEN, MADE IN ORDER TO DETERMINE THE SPECIFIC GRAVITY OF THE ROCKS COMPOSING THAT MOUNTAIN. (*Philosophical Transactions*, 1811, pp. 347–378.)

A careful survey of the dispcsition and specific gravities of the strata round Schehallien, and an estimate of the correction to be applied to Hutton's result, leading to limits 4·5588 and 4·867 between which the mean density should lie.

10. 1814. **ZACH (BARON DE).** L'ATTRACTION DES MONTAGNES ET SES EFFETS SUR LES FILS À PLOMB, ETC., AVIGNON.

A determination of the deviation of the plumb-line by the mountain Mimet, some miles north of Marseilles, the second station being an island nine miles S.W. of Marseilles. The instrument used was probably inadequate for the purpose, and no attempt was made to deduce the mean density.

11. 1820. **LAPLACE (M. DE).** SUR LA DENSITE MOYENNE DE LA TERRE. (*Ann. de Chem. et de Phys.*, xiv., 1820, pp. 410–417. *Tilloch Phil. Mag.*, lvi. pp. 322–326.)

Only a brief account of the results found by experiment up to the time of writing, and a reference to his own work as showing the heterogeneity of the earth.

12. 1821. **HUTTON (C.).** ON THE MEAN DENSITY OF THE EARTH. (*Philosophical Transactions*, 1821, pp. 276–292.)

A résumé of the Schehallien experiment ; a vindication of his claim to have taken an important part in it ; and a criticism of Cavendish's calculations. The errors he thought he detected were later shown by Baily either to be non-existent or unimportant. (*Mem. Ast. Soc.*, xiv., No. 19 below.) His final value from the Schehallien experiment is given as 4·95.

13. 1824. **CARLINI (FRANCESCO).** OSSERVAZIONE DELLA LUNGHEZZA DEL PENDOLO SIMPLICE FATTE ALL ALTEZZA DI MILLETISE SUL LIVELLO DEL MARE. (*Milano Effem. Astr.* 1824, pp. 28–40. An account of Carlini's experiments by Sabine, *Quarterly Journal of Science*, ii., 1827, pp. 153–159.)

Carlini found the length of the seconds pendulum at the Hospice on Mount Cenis, 1943 metres above sea-level, to be ·210 mm. greater than it would have been without the mountain at the same altitude and latitude. Assuming a certain shape and density for the mountain, the value of the earth's mean density is 4·39. Sabine gives the result as ·0076 inches longer than it would have been without the mountain at the same altitude, and deduces a value 4·77. See No. 18, below.

14. **1826–8.** ACCOUNT OF EXPERIMENTS MADE AT DOLCOATH MINE IN CORNWALL, IN 1826 AND 1828, FOR THE PURPOSE OF DETERMINING THE DENSITY OF THE EARTH. 8vo, pp. 16, Cambridge, 1828.

Privately printed and issued with a request that it should not be published. Early attempts by the method described in No. 22.

Accounts of these experiments are fully given in

15. **DROBISCH (M. W.).** UEBER DIE IN DEN MINEN VON DOLCOATH IN CORNWALL NEUERLICH ANGESTELLTEN PENDEL-BEOBACHTUNGEN. (*Pogg. Ann.*, x. 1827, pp. 444–456.)

16. **DROBISCH (M. W.)** AUSFÜHRLICHER BERICHT UEBER MEHRERE IN DEN JAHREN 1825 UND 1826 IN DEN MINEN VON DOLCOATH ZUR BESTIMMUNG DER MITTLEREN DICHTIGKEIT DER ERDE ANGESTELLTE PENDEL-VERSUCHE. (*Pogg. Ann.*, xiv., 1828, pp. 409–426.

17. **1837. REICH (F.)** VERSUCHE ÜBER DIE MITTLERE DICHTIGKEIT DER ERDE MITTELST DER DREHWAGE. 8vo. Freiberg, 1838.

Some account of the experiment is given in a note, "Sur la densité de la Terre," *Comptes Rendus*, v. 1837, pp. 697–700. Referred to in Baily, *Mem. Ast. Soc.* 14, No. 19 below.

A repetition of Cavendish's experiment, with smaller spheres and some changes of method. Result 5·44, afterwards corrected to 5·49.

18. **1841. GIULIO (CARLO IGNAZ).** SUR LA DÉTERMINATION DE LA DENSITÉ MOYENNE DE LA TERRE DÉDUITE DE L'OBSERVATION DU PENDULE FAITE A L'HOSPICE DU MONT CENIS, PAR M. CARLINI, EN SEPT. 1821. (*Torino Mem. Accad.*, iii., 1841, p. 379.)

I have not seen this paper, or any account of its contents, beyond a statement by v. Sterneck that the resulting mean density is 4·950.

19. **1842. BAILY (FRANCIS).** AN ACCOUNT OF SOME EXPERIMENTS WITH THE TORSION-ROD FOR DETERMINING THE MEAN DENSITY OF THE EARTH. (*Memoirs of the Royal Astronomical Society*, vol. xiv. pp. 1–120 and i–ccxlviii.)

Baily's repetition of Cavendish's experiment. A large number of determinations with different attracted masses. Mean value 5·6747.

20\. 1847. **HEARN (G. W.).** ON THE CAUSE OF THE DISCREPANCIES OBSERVED BY MR. BAILY WITH THE CAVENDISH APPARATUS FOR DETERMINING THE MEAN DENSITY OF THE EARTH. (*Philosophical Transactions*, 1847, pp. 217–230.)

A curious attempt to explain the anomalies in Baily's observations by supposing that there was a magnetic action between the masses. The polarity of the attracting masses was supposed to be reversed on changing their position, so that an attraction on one side became a repulsion on the other, and practically only the variations in the amount of polarity would be detected.

These variations he supposes to be somewhat irregular. The total magnetic action he makes out in some cases to be far larger than the gravitative action. He was evidently unaware of Cavendish's experiments on the action of magnets on the balls, or of Reich's with a cast-iron mass.

21\. 1852. **REICH (F.).** NEUE VERSUCHE MIT DER DREHWAAGE. (*Leipzig. Abh. Math. Phys.* i., 1852, pp. 383–430.) NEUE VERSUCHE ÜBER DIE MITTLERE DICHTIGKEIT DER ERDE. (*Pogg. Ann.*, lxxxv., 1852, pp. 189–198; *Phil. Mag.* v. 1853, pp. 154–159.)

After explaining that he has revised his former results, using Baily's method and altering the final result from 5·45 to 5·49, he describes a more extensive series giving result 5·5832. Some interesting experiments were made with attracted masses of bismuth and iron. The former gave nearly the same result, the latter a higher one. But in any case, the magnetic action in the original experiment must be negligible.

22\. 1856. **AIRY (G. B., Astronomer Royal).** ACCOUNT OF PENDULUM-EXPERIMENTS UNDERTAKEN IN THE HARTON COLLIERY FOR THE PURPOSE OF DETERMINING THE MEAN DENSITY OF THE EARTH. (*Philosophical Transactions*, 1856, pp. 297–342.)

A full account of the experiments, survey, and calculations for the determination of the ratio of gravity above to gravity a known depth below the surface. Value of the mean density 6·565.

23\. 1856. **AIRY (G. B.).** SUPPLEMENT TO THE "ACCOUNT OF PENDULUM-EXPERIMENTS. . . ." (the paper above). (*Philosophical Transactions*, 1856, pp. 343–355.)

An acount of experiments undertaken at Greenwich to determine the temperature correction for the pendulums, with an addendum by Prof. Stokes in reference to the effect of the earth's rotation and ellipticity.

24\. 1856. **JAMES (Lieut.-Col. R.E., F.R.S) AND CLARKE (Capt. R.E.).** ON THE DEFLECTION OF THE PLUMB-LINE AT ARTHUR'S SEAT AND THE MEAN SPECIFIC GRAVITY OF THE EARTH. (*Philosophical Transactions*, 1856, pp. 591–606.)

In the course of the survey round Edinburgh, a determination of the deflection of the plumb-line by Arthur's Seat was made by means of stellar zenith distances.

The paper contains a discussion of the mode of treating the astronomical observations and a full account of the mode of computing the attraction at each station of the surrounding country by Hutton's method. The value of mean density is 5·316. An inquiry is added into the reason for the general deflection of about 5″ in the neighbourhood of Edinburgh.

A fuller account of the data is given in the "Account of the Observations and Calculations of the Principal Triangulation" of the Ordnance Survey 1858.

25. 1873. **CORNU (A.)** ET **BAILLE (J.).** DÉTERMINATION NOUVELLE DE LA CONSTANTE DE L'ATTRACTION ET DE LA DENSITÉ MOYENNE DE LA TERRE. (*Comptes Rendus*, vol. lxxvi., 1873, pp. 954–8.)

A brief account of the apparatus used in a new determination by the torsion balance method and a statement of the mean results obtained so far, viz., 5·50 and 5·56.

26. 1878. **CORNU** ET **BAILLE.** ÉTUDE DE LA RÉSISTANCE DE L'AIR DANS LA BALANCE DE TORSION. (*Comptes Rendus*, vol. lxxxvi., 1878, pp. 571–4.)

An account of some observations to determine whether the resistance of the air is accurately proportional to the velocity. If so, the successive swings should be in exact geometric progression and so they were found to be with great accuracy.

27. 1878. **CORNU** ET **BAILLE.** SUR LA MESURE DE LA DENSITÉ MOYENNE DE LA TERRE. (*Comptes Rendus*, vol. lxxxvi. pp. 699–702.)

A notice of addition of two more spheres of mercury and a reduction of distance from the torsion rod, the force being thus quadrupled. The time of vibration has remained at 408 seconds for more than a year. The results are still 5·56.

An error is pointed out in Baily's method in which he takes the last elongation in one postion as the first in the next, notwithstanding the movement of the masses meanwhile. Rejecting the first elongation in each position as vitiated, Baily's results are appreciably reduced, and the estimate, founded on some random examples, is that the mean would be lowered from 5·68 to 5·55.

28. 1878. **CORNU** ET **BAILLE.** INFLUENCE DES TERMES PROPORTIONNELS AU CARRÉ DES ÉCARTS DANS LE MOUVEMENT OSCILLATOIRE DE LA BALANCE DE TORSION. (*Comptes Rendus*, vol. lxxxvi. pp. 1001–4.)

An examination of the effect of terms higher than the first in the restoring force.

Taking $\ddot{\omega} + h\,\dot{\omega} + s\,\omega = W\,\alpha^2$

and neglecting h^2

$$\omega = A\,e^{-\alpha t} \sin \frac{2\pi(t - t_0)}{T} + \frac{W A^2 e^{-2\alpha t}}{3s} \left\{1 + \cos^2 \frac{2\pi(t - t_0)}{T}\right\}.$$

$$\text{where } \alpha = \frac{h}{2} \text{ and } \frac{2\pi}{T} = \sqrt{s - \alpha^2}$$

Hence, a term of half the period is superposed on the solution obtained by neglecting $W\omega^2$.

It follows that neither the point through which the rod passes at equal intervals of time nor that about which the successive vibrations have the same ratio is the true position of rest.

29. 1878. **JOLLY** (Ph. von). DIE ANWENDUNG DER WAAGE AUF PROBLEME DER GRAVITATION. (*Abhand. der k. Bayer. Akad. der Wiss.*, 2 cl. xiii. Bd. 1 Abth. pp. 157–176; *Wied. Ann.*, vol. v. pp. 112–134.)

Account of experiments made with a balance to determine the variation of weight with height. A kilogramme was weighed in one pan of a balance and then in a pan suspended by a wire from the first and about 5 metres below it. An increase, nearly that calculated by the ordinary theory, was observed. The author suggests that if the experiment were repeated under suitable conditions with a large mass of lead under the lower position, the increase in the gain of weight would measure the attraction of the lead and so give the mean density of the earth.

30. 1878. **POYNTING** (J. H.). ON A METHOD OF USING THE BALANCE WITH GREAT DELICACY, AND ON ITS EMPLOYMENT TO DETERMINE THE MEAN DENSITY OF THE EARTH. (*Proceedings of the Royal Society*, vol. xxviii., 1878, pp. 2–35.)

An account of preliminary experiments to test a method of determining the mean density by putting a lead sphere 1 foot in diameter under the weight hung by a wire from one arm of a balance, The increase in weight gave the attraction and thence the mean density was calculated to be 5·69, but with large deviations from the mean.

31. 1880. **MENDENHALL** (T. C.). DETERMINATION OF THE ACCELERATION DUE TO THE FORCE OF GRAVITY AT TOKIO, JAPAN. (*American Journal of Science*, vol. xx. pp. 124–132.)

Contains an account of the method used in the experiment described below to determine the time of vibration of the pendulum.

32. 1881. **MENDENHALL** (T. C.). ON A DETERMINATION OF

THE FORCE OF GRAVITY AT THE SUMMIT OF FUJIYAMA, JAPAN. (*American Journal of Science*, vol. xxi. pp. 99–103.)

A determination of the relative values of gravity at Tokio and on the summit of Fujiyama, which is a very symmetrically shaped cone. The mean density deduced 5·77.

33. 1881. **JOLLY (Ph. v.).** DIE ANWENDUNG DER WAAGE AUF PROBLEME DER GRAVITATION. Zweite Abhandlung. (*Abh. d. k. Bayer. Akad. d. Wiss.*, 2 cl. 14 Bd. 2 Abth. pp. 26. *Wied. Ann.*, xiv. 1881, pp. 331–355.)

Experiments carried out in accordance with the suggestion at the close of the previous paper. A balance was fixed at the top of a tower. From its scale pans were hung two other scale-pans by wires 21 metres below. The gain in weight of a glass vessel filled with 5 kg. of mercury when moved from the upper to the lower pan on one side of the balance was measured, and then a large lead sphere a metre in diameter was built up under the lower sphere, and a gain greater by ½ mgm. was observed. From this the mean density of the earth was calculated to be 5·692.

34. 1882. **STERNECK (R. von).** UNTERSUCHUNGEN ÜBER DIE SCHWERE IM INNERN DER ERDE, AUSGEFÜHRT IM JAHRE 1882 IN DEM 1000 METER TIEFEN ADALBERT-SCHACHTE DES SILBERBERGWERKES ZU PRIBRAM IN BÖHMEN. (*Mittheilungen des k. u. k. Militar. geogr. Inst. zu Wien*, ii., 1882, pp. 77–120.)

An invariable half-second pendulum was swung at the surface and at two different depths in the mine, being compared with a half-second clock rated from a standard clock at the surface by means of a pocket chronometer. The value of gravity at the underground stations, one 456·5 metres below the other, was practically the same. The values for the mean density given by comparison of each lower station with the surface are 6·28 and 5·01.

35. 1883. **STERNECK (R. von).** WIEDERHOLUNG DER UNTERSUCHUNGEN ÜBER DIE SCHWERE IM INNERN DER ERDE, AUSGEFÜHRT IM JAHRE 1883 IN DEM 1000 METER TIEFEN ADALBERTSCHACTE DES SILBERBERGWERKES ZU PRIBRAM IN BÖHMEN. (*Mitth. des k. u. k. Mil. geogr. Inst. zu Wien*, iii., 1883, pp. 59–94.)

A repetition of the experiments described in the previous paper, with the addition of two more underground stations and greatly improved pendulum observations. Two invariable half-second pendulums were used, one at the surface, the other at one of the four stations at different depths in the mine, the pendulums being interchanged at each pair of stations to complete a determination. The coincidences were observed at the same time at the two stations and in comparison with the same clock, which gave half-second electric signals at the two stations. The mean result for the density of the earth is 5·68, but the value increases with the depth of the lower station.

36. 1883. **BAILLE (J. B.).** ÉTUDES SUR LA RÉSISTANCE DE L'AIR DANS LES MOUVEMENTS OSCILLATOIRES TRÈS LENTS. (*Comptes Rendus*, 1883, vol. xcvi. pp. 1493–5.)

A study by the torsion balance of the variations of resistance with form and dimensions of the moving body and with temperature and pressure of the air.

The resistance per unit area diminishes with increase of surface normal to the motion; it varies with the dimensions and shape of the case, and also varies with temperature and pressure, especially at high pressure. At fractions of an atmosphere the variation is small.

37. 1884. **KÖNIG (A.)** AND **RICHARZ (F.).** EINE NEUE METHODE ZUR BESTIMMUNG DER GRAVITATIONS-CONSTANTE. (*Sitzungsberichte der Berl. Akad.*, 1884, p. 1203. *Wied. Ann.*, xxiv., 1885, p. 664. *Nature*, vol. xxxi. p. 484.)

In the last-named place is an account of the proposed method, forwarded by the authors to correct the incomplete description in early notices in the same volume.

In principle the experiment which is now (1892) in progress may be regarded as a development of Jolly's experiment. A balance is mounted on a large block of lead with perforations below the scale pans. The right weight is first above and the left weight below the block; then the right weight is below and the left above. The difference between the two weighings, when corrected for variation in altitude, is four times the attraction of the block.

38. 1884. **HICKS (W. M.).** ON SOME IRREGULARITIES IN THE VALUES OF THE MEAN DENSITY OF THE EARTH, AS DETERMINED BY BAILY. (*Proc. of Cambridge Phil. Soc.*, vol. v. part 2, 1884, pp. 156–161.)

The author points out that if Baily's results be arranged in order of temperature there is a steady fall in value of the mean density with rise of temperature.

39. 1885. **STERNECK (R. von).** UNTERSUCHUNGEN ÜBER DIE SCHWERE IM INNERN DER ERDE, AUSGEFÜHRT IM JAHRE 1885 IN DEM ABRAHAMSCHACTE DES SILBERBERGWERKES "HIMMELFAHRTSFUNDGRUBE" BEI FREIBERG IN SACHSEN. (*Mitth. des k. u. k. Mil. geog. Inst. zu Wien*, vi. 23 pp.)

Professor Bruhns had found a decrease in gravity in this mine below the surface. Major von Sterneck, therefore, determined to reinvestigate the value with his invariable pendulums, using the two underground stations selected by Bruhns and two others. He found an increase of value with increase of depth but much more rapid than at Pribram. The increase with increase of temperature was, however, nearly the same in both cases, a result at present unexplained. The values of the mean density of the earth were much higher than before, the mean being over 7.

40. 1885. **WILSING (J.).** BESTIMMUNG DER MITTLEREN DICHTIGKEIT DER ERDE MIT HÜLFE EINES PENDEL-APPARATES. (*Publica-*

tionen des Astro-Physikalischen Observatoriums zu Potsdam, No. 22, vi. Band, 2 Stück, pp. 35–127.)

A pendulum rod, with nearly equal balls at the upper and lower ends, is supported by a knife-edge just above its centre of gravity so that the gravity restoring couple is small and the time of swing great. Attracting masses are brought up one on one side of the upper ball, the other on the other side of the lower ball, and the deflection is observed. From the time of swing the attraction can be calculated, and the mean density of the earth is deduced as 5·594.

41. 1889. **WILSING (J.).** BESTIMMUNG DER MITTLEREN DICHTIGKEIT DER ERDE MIT HÜLFE EINES PENDEL-APPARATES (Zweite Abhandlung). (*Publ. des Astro.-Phys. Observ. zu Potsdam*, No. 23, vi. Band, 3 Stück, pp. 133–191.)

An account of further experiments by the pendulum balance described in the previous paper. Various improvements are introduced, especially some to guard against temperature changes.

The result is now 5·579.

42. 1889. **BOYS (C. V., F.R.S.).** ON THE CAVENDISH EXPERIMENT. (*Proceedings of the Royal Society*, vol. xlvi. pp. 253–268.)

A discussion of the best dimensions for the torsion apparatus and a description of a small instrument which, with half-inch beam and small attracting masses, gives a large deflection when the period of vibration is only 80 seconds. The author states his intention to construct apparatus for absolute measurements of the gravitation constant.

43. 1889. **LASKA (W.).** UEBER EINEN NEUEN APPARAT ZUR BESTIMMUNG DER ERDDICHTE. (*Zeitschrift für Instrumenten Kunde*, 1889, vol. ix. pp. 354–5.)

An account of apparatus prepared to carry out a pendulum-deflection method after the method of Wilsing, the position of the pendulum being indicated by interference bands between a glass plate on its upper end and a nearly parallel fixed plate, after the manner of Fizeau's expansion apparatus. The attracting mass to be a sphere of mercury.

No results given.

44. 1892. **POYNTING (J. H.).** ON A DETERMINATION OF THE MEAN DENSITY OF THE EARTH AND THE GRAVITATION CONSTANT BY MEANS OF THE COMMON BALANCE. (*Philosophical Transactions*, vol. clxxxii. (1891), A, pp. 565–656.)

The Paper is reprinted as the Second Part of this Essay.

45. 1893. **BERGET (A.).** DÉTERMINATION EXPÉRIMENTALE DE LA CONSTANTE DE L'ATTRACTION UNIVERSELLE, AINSI QUE DE LA

MASSE ET DE LA DENSITÉ DE LA TERRE. (*Comptes Rendus*, vol. cxvi. pp. 1501–3.)

Above the surface of a lake 32 hectares (79 acres) in area, at Habay-la-Neuve, in Belgian Luxemburg, was fixed a hydrogen gravimeter, or a barometer with the cistern closed and containing hydrogen to act as a spring to support the column of mercury. The level of the lake could be varied, and as it fell the mercury in the gravimeter became lighter, and the hydrogen supported a longer column. The position of the column was determined by interference fringes, formed between the surface of the mercury in the vacuum and the end of the tube which was an optically worked plane. The level of the lake was lowered in two stages of 50 cm. each, and then raised again in equal stages. The column of mercury rose $63/10^8$ cm. and $126/10^8$ cm., and then fell equal amounts within 2°/o. From these observations the mean density of the earth is calculated to be 5·41. In *Comptes Rendus*, vol. cxvii. pp. 96–7, M. Gouy points out that the accuracy claimed implies a temperature constant within a five-millionth of a degree during the several hours occupied by the experiment, a constancy utterly beyond anything as yet considered possible.

The two following papers appear to contain accounts of early attempts to measure variation of gravity with variation of distance from the earth's centre. I have not seen the papers, and know nothing as to their quality, though the result obtained by Bertier seems to indicate a want of exactness.

1773. **BERTIER (Le Père J. C.).** LETTRE DU P. BERTIER DE L'ORATOIRE, OÙ IL EXPOSE UNE EXPÉRIENCE QUI TEND À PROUVER QUE LES CORPS PESENT D'AUTANT PLUS QU'ILS SONT PLUS ÉLÉVES SUR LA TERRE JUSQU'À UNE PETITE DISTANCE. (*Journal de Physique*, ii., 1773, pp. 251 and 275.)

WOLF (*Mémoires rél. à la Phys.*, 4. B. 55) says that this experiment was the same as Jolly's first experiment. Other letters and memoirs on the subject are in vol. v. of the *Journal de Physique*.

1775. **DOLOMIEU (De).** EXPÉRIENCES SUR LA PESANTEUR DES CORPS À DIFFÉRENTES DISTANCES DU CENTRE DE LA TERRE, FAITES AUX MINES DE MONTRELAY EN BRETAGNE. (*Journal de Physique*, 6 Juillet 1775.)

THE MEAN DENSITY OF THE EARTH.

PART I.

HISTORICAL.

INTRODUCTORY DISCUSSION OF THE NATURE OF THE INVESTIGATION.

THE determination of the mean density of the earth may be regarded also as the determination of the complete expression of Newton's law of gravitation. A brief consideration of the facts set forth by Newton in the third book of the "Principia" will show this most clearly, and will at the same time suggest modes in which the determination should be made.

Starting with any single planet—say the earth—and referring its position to a system, fixed relative to the sun and the distant stars, direct astronomical observation shows that it may be described to a close approximation, as moving in an ellipse with the sun in one focus and at such speed that the line from the centre of the sun to the centre of the planet sweeps out equal areas in equal times. This implies, as Newton showed, that the acceleration of the planet is towards the sun and inversely as the square of its distance from that body.

Now comparing the different planets, observation shows that $(\text{length of year})^2 / (\text{mean distance})^3$ is the same for each, and from this it follows that the constant of acceleration is the same for all, or that at the unit distance they would all have the same acceleration if the law holding for each in its own orbit held for it at all distances.

So far this is mere time-geometry, or a description of position and rate of change of position, and we might have other equally true, if less convenient, modes of description referred to other standards, such as the epicyclic geocentric mode of the ancients, or the practical mode in common use in which the co-ordinates of a planet are measured with regard to some observatory, its meridian, and horizon.

But if we regard the accelerations as indicating forces the different methods of description are no longer equivalent. We must select that which gives a system of forces most consistent in itself and most in accord with our terrestrial experience. Here the heliocentric method, with the modification described hereafter, is immensely superior to any other, and, adopting it, we must suppose that the accelerations of the planets indicate forces towards the sun, and since the constant of acceleration is the same for all, that the forces on equal masses are inversely as their distances squared from the sun, whatever planets the masses belong to. In other words, the sun has no favourite among its attendants, but pulls on each pound of each according to the same rule.

But the assumption that the accelerations indicate forces of the kind we experience on the earth, carries with it the supposition of equality of action and reaction, and so we conclude that each planet reacts on the sun with a force equal and opposite to that exerted by the sun on the planet. Hence, each acts with a force proportional to its own mass, and inversely as the square of its distance away. If we suppose that there is nothing special in the attraction of the sun beyond great magnitude corresponding to great mass, we must conclude that the sun also acts with a force proportional to its mass. But we have just shown that the force is proportional to the mass acted on. Hence, we have the force proportional to mass of sun × mass of planet / $(\text{distance apart})^2$.

Now, turning to any of the smaller systems consisting of a primary and its satellites, the shape of orbit and the motion of the satellites agree with the supposition that the primary is acting with a force according to the inverse square law. It is important for our special problem to note here that in the case of the earth we must include in the term satellite any body at its surface which can be weighed or moved.

We are therefore led to conclude that the law is general, or that if we have any two bodies, of masses M_1 and M_2, d apart, the force on either is

$$\frac{G\,M_1\,M_2}{d^2},$$

where G is a constant, the constant of gravitation.

The acceleration of one of them, say M_2, towards the other is

$$\frac{G\,M_1}{d^2}.$$

If this conclusion is accepted, we can at once determine the *masses* of the various primaries in terms of that of the sun for acceleration of satellite towards primary

$$= \mathrm{G}. \quad \text{Mass of primary} / (\text{distance of satellite})^2$$

and acceleration of primary towards sun

$$= \mathrm{G}. \quad \text{Mass of sun} / (\text{distance of primary})^2.$$

By division G is eliminated, and we obtain the ratio of the masses in terms of quantities which may be measured by observation.

A confirmation of the generality of the law is obtained from the perturbations of the planets from the elliptic orbits which we have for simplicity supposed them to describe.

These perturbations, in any one planet, can at least approximately be analysed into separate disturbances, each due to one of its fellow planets, acting with a force inversely as the square of its distance away, and if we assume this force proportional to the mass of the disturber we obtain another measure for this mass in terms of that of the sun.

The concordance of the two methods is as complete as we could expect.

The determination of the masses of the different members of our system in terms of that of the sun, enables us to choose a still more satisfactory origin for our system of reference than the centre of the sun—viz., the centre of mass of the whole system. The change is small, but without it we could not account for all the motions merely by a set of inverse square forces in which action and reaction were equal and opposite.

We have for simplicity considered the sun and planets as without appreciable dimensions as compared with their distances apart. But measurement shows that they are all approximately spheres, and the attraction on a sphere with density varying only with the distance from the centre, *i.e.*, consisting of homogeneous concentric shells, if it is considered as the resultant of the attractions on the separate particles, all according to the same inverse square law, is the same as that on the whole mass collected at the centre of the sphere. Further, if the attraction is due, not to the attracting body as a whole but to its separate parts, each acting, as it were, independently and according to the same law, then an attracting sphere acts as if it were all concentrated at its centre. Since the planets, to a near approximation, behave as if they were merely concentrated masses at their centres, and since the deviations from this behaviour, such as the earth's precession, can all be accounted for by their departure from sphericity, we have strong presumption that the attraction is really the resultant of all the attractions, each element m_1 of one body acting on each element m_2 of the other with force $\mathrm{G}m_1m_2/d^2$.

Astronomical observation enables us, then, to compare the masses of the various members of the solar system, with each other or even to make a table of specific gravities, choosing any one as the standard substance. Thus, if we take the earth as standard, the mean specific gravity of the sun is about 0·25, that of Mercury about 1·25, that of Venus and Mars about 0·9, and so on.

But this does not give us any idea of the specific gravity in terms of known terrestrial substances or any idea of the masses in terms of the terrestrial standards, the kilogramme or the pound. It is true that Newton, with little more than the astronomical data at his command, made a celebrated guess on the specific gravity of the earth in terms of water, which runs thus in Motte's translation of the "Principia" (vol. ii. p. 230, ed. 1729, book iii. prop. x.): "But that our globe of earth is of greater density than it would be if the whole consisted of water only, I thus make out. If the whole consisted of water only, whatever was of less density than water, because of its less specific gravity, would emerge and float above. And upon this account, if a globe of terrestrial matter, covered on all sides with water, was less dense than water, it would emerge somewhere; and the subsiding water falling back, would be gathered to the opposite side. And such is the condition of our Earth, which, in great measure, is covered with seas. The Earth, if it was not for its greater density, would emerge from the seas, and according to its degree of levity, would be raised more or less above their surface, the water and the seas flowing backwards to the opposite side. By the same argument, the spots of the sun which float upon the lucid matter thereof, are lighter than that matter. And however the Planets have been form'd while they were yet in fluid masses, all the heavier matter subsided to the centre. Since, therefore, the common matter of our Earth on the surface thereof, is about twice as heavy as water, and a little lower, in mines is found about three or four, or even five times more heavy; it is probable that the quantity of the whole matter of the Earth may be five or six times greater than if it consisted all of water, especially since I have before shewed that the Earth is about four times more dense than Jupiter."

It is not a little remarkable that Newton hit upon the limits between which the values found by subsequent researches have nearly all laid.

In order, then, to complete the expression of the law of gravitation, we must connect the celestial with the terrestrial scale of densities. In fact, we must do for the masses of the solar system that which we do for their distances in the determination of the solar parallax, though we cannot proceed quite so directly in the former case as in the latter in connecting the celestial and terrestrial measures. If we could measure the acceleration, say, of the moon, due to any terrestrial body of known shape and density; if, for instance, we knew the form and extent of our tidal wave and its full lunar effect, we could at once find the mass of the earth in terms of that of the wave, or its density as compared with sea-water.

But at present this cannot be done with any approach to accuracy, and the only method of solving the problem consists in finding the attraction between two bodies on the earth of known masses a known distance apart, and comparing this with the attraction of the earth on a known mass at its surface instead of its attraction as a heavenly body. Since the law of attraction is by observation the same at the surface of the earth and at a distance, we can thus find the mass of the earth in terms of either of the known masses.

To take an illustration from an experiment hereafter described, let us suppose that a spherical mass of 20 kilos. is attracted by another spherical mass of 150 kilos. when the centres are 30 cm. apart with a force equal to the weight of $\frac{1}{4}$ mgm. or $\frac{1}{80000000}$ of the weight of the 20 kilos when the latter is on the surface of the earth and 6×10^8 cm. from its centre, we have:

$$\frac{\text{Mass of Earth}}{(6 \times 10^8)^2} : \frac{150{,}000}{30^2} = 1 : \frac{1}{80000000}$$

whence mass of earth $= 5 \times 10^{27}$ grammes nearly.

The volume of the earth is about 9×10^{26} cc., whence the mean density of the earth is about 5·5.

Or, using the experiment to give the constant of attraction, and expressing the masses in grammes, the weight of $\frac{1}{4}$ mgm. or

$$\cdot 00025g = \frac{G \times 150{,}000 \times 20{,}000}{30^2}$$

Whence if $g = 981$,

$$G = \frac{981 \times \cdot 00025 \times 30^2}{150{,}000 \times 20{,}000}$$

$$= \frac{7}{10^8} \text{ nearly.}$$

A determination of G completes the expression of the law of gravitation.

The Methods of Experiment.

These naturally fall into two classes. In the one class some natural mass is selected, either a mountain or a part of the earth's crust, and its mass and form are more or less accurately determined by surveys and mineralogical examination. Its attraction on a plumb-bob at one side, or on a pendulum above or below it is then compared with the attraction of the whole earth on the same body.

In the other, the laboratory class of experiment, a smaller mass, such as may be easily handled, is placed so as to attract some small suspended body, and this attraction is measured and compared with the attraction of the earth for the same body.

Suppositions as to the Qualities of Gravitation.

In all the methods certain suppositions are made as to the qualities of gravitation, which are probably justified either by the results of the experiments themselves or by other experience.

The following is a list of these suppositions with their justifications:

1. That the law of gravitation holds throughout the long range from interplanetary distances down to the distances between the attracting bodies in the laboratory experiments.

The first step in the descent from celestial spaces is justified by the fact that the acceleration of gravity at the earth's surface agrees with its value on the moon, as attracted by the earth. The further step downward appears to be justified by the fair agreement of the results obtained by the various forms of Cavendish, balance, and pendulum experiments on the mean density—experiments which have been conducted at distances varying from feet down to inches. Where the law ceases to hold is yet a matter for experiment to determine. When bodies come into what we term contact, the adhesion may possibly still be due to gravitation, according to the inverse square law, though the varying nature of the adhesion in different cases seems to point to a change in the law at such minute distances.

2. That gravitation is not selective like electric and magnetic action.

The agreement of astronomical observations with deductions from the law is hardly a sufficient proof of this supposition. For suppose that some matter were attracted more than in proportion to its mass and some less, there might be in all the planets such quantities of the two kinds that the same general average is maintained.

This may be an extravagant supposition, and yet, if we are to adopt some of the current ideas regarding comets and their tails, we are getting very near it. But with regard to ordinary terrestrial matter, Newton's hollow pendulum experiments ("Principia," lib. iii. prop. vi.) repeated with more detail and precision by Bessel ("Versuche über die Kraft, mit welcher die Erde Körper von verschiedene Beschaffenheit anzieht," Abhand. der Berl. Ak. 1830, p. 41; or "Mémoires relatifs à la Physique," tome v. pp. 72–133) are conclusive that the earth as a whole is not selective. Even here, however, the result might be due to an average of equal excesses and defects. But again we may quote the various mean density experiments and especially those made by Bailey, in which a number of different attracting and attracted substances have been used with nearly the same results.

3. There is no varying quality in the medium intervening between the attracting masses corresponding to electric specific inductive capacity or magnetic permeability.

Astronomical observations are not decisive, for the medium intervening between the sun and planets, approaches a vacuum where so far we have no evidence for variation in quality even for electric and magnetic induction. In the case of the earth too, its spherical form might render observation inconclusive, for just as a sphere composed of concentric dielectric shells each with its surface uniformly electrified would have the same external field in air, whatever the dielectric constant, if the quantity of electrification within were the same, so the earth might have the same field in air whatever the varying quality of the underlying strata as regards the transmission of the action across them if they were only suitably arranged.

But the fact that there is nothing like refraction of the lines of gravitative force corresponding to the tangent law of refraction for electric and magnetic lines of induction appears decisive. If further evidence be needed it may be pointed out that again the different mean density experiments come to our aid. Various forms of case of various material have been used to separate the attracting and attracted bodies, with no great effect on the result.

Further, different forms of attracting masses have been used, the cylinders of Dr. Wilsing giving nearly the same result as the spheres of other workers. The field due to a cylinder would almost certainly depend on its gravitative permeability.

4. There is nothing of a directive nature in the gravitative field.

It is possible to imagine that, superposed on the ordinary field, there is a directive one, which might manifest itself in certain cases. Thus, an elongated body might tend to place its longer axis vertical on the earth's surface. Nothing of the kind has, however, been detected, and for ordinary bodies we may dismiss the supposition. But there is one case which appears to require further experimental investigation, that of crystalline structure with different properties along different axes. Thus, the attraction between two crystal spheres might be different for a given distance according as their like axes were parallel or crossed.

If such difference existed it should show itself by a directive action on one sphere in the field of the other, For allow one sphere to approach the other with the axes arranged in such way that there is a maximum pull, then revolve the sphere brought up into the position of minimum pull, then withdraw it. A surplus of work remains over unless work has been done in the revolution, *i.e.*, unless there is a couple tending to keep the sphere with its axes in the position of maximum pull. We can hardly imagine any other way in which the surplus could disappear, so that we must suppose a directive action if there is a difference of pull for different positions of the axes.

I propose to investigate this experimentally by means of two quartz spheres, respectively about 1 and 2½ inches diameter. The smaller one will be suspended by a quartz fibre, but as its weight (25 grms.) would require a rather strong fibre to support it, it will be suspended in water and so nearly buoyed up by a sort of small Nicholson's hydrometer that it only just sinks. To this the fibre will be attached; the fibre will then only need to be strong enough to bear the difference between the weight of the sphere and the upward pull of the buoy. A mirror attached to the buoy will indicate in the usual way, by telescope and scale, the azimuth of the sphere.

Outside the water vessel the larger sphere will be placed and turned about its centre in different directions.

I have already made some experiments, without success, to test the existence of a difference of direct pull for different axial directions by suspending the small sphere and buoy at the end of a torsion arm, but the small torsion of the fibre was quite unable to keep the rod from turning round, under the disturbing action of convection currents. The buoy, sooner or later, came against the side of the vessel and here the adhesion prevented return. While planning a reconstruction of the apparatus, it occurred to me that there should be a directive couple, at least of the order of the difference of the moments of the direct pulls at the end of the torsion rod, and a rough calculation on assumed data confirms this. I am now arranging the apparatus to test its existence.*

But even should such action exist, it would hardly affect the conclusions drawn from mean density experiments, for in these there has been at least no regularity of crystalline structure, and in some cases, as in the mercury used by Cornu and Baille no question of crystalline structure at all.

5. That the density obtained from the formula used to express the attraction of the earth is the real mean density, *i.e.* $\frac{\int \rho \, dv}{\int dv}$.

In the calculation of the mean density the earth is replaced by

---

* Since the above was written the suspension in a liquid by the aid of a buoy has been abandoned, for the convection currents, even with the central position of the mass, produced enormous disturbances. The apparatus is now being prepared with a much smaller suspended crystal in air.

an ellipsoid of the same form as nearly as measurement can determine it, and of uniform density, and the result actually obtained is the value of such uniform density, as would produce the observed value of gravity. But though the density of the earth undoubtedly varies, increasing at least at first as we approach the centre, "it is probable that the earth consists approximately of spherical strata of equal density. Any material deviation from this arrangement could hardly fail to produce an irregularity in the variation of gravity and consequently in the form of the surface, since we know that the surface is one of equilibrium. Hence, we may assume, when not directly considering the ellipticity, that the density ρ is a function of the distance from the centre." (Stokes' "Math. Papers," vol. ii. p. 122.) With this assumption the masses of the actual and of the hypothetical uniform earth must be the same, and the uniform density of the latter must be the mean density of the former. And indeed without any assumption the fact that gravity at the earth's surface and at the moon may both be expressed in terms of the same uniformly dense spheroid and the distances, shows that the two densities are the same, for the actual and the hypothetical earth must have the same mass to have the same attraction at the distance of the moon.

HISTORICAL ACCOUNT.

Newton's Calculations of the Attraction between Terrestrial Bodies.

From a passage in his treatise "De Mundi Systemate" (*Opera*, ed. Horsely, vol. iii. p. 194, § 22), it is evident that Newton considered the possibility of measuring the attraction between terrestrial masses, for he says: "The experiments in terrestrial bodies come to no account; for the attraction of homogeneous spheres near their surfaces are as their diameters. Whence, a sphere of one foot in diameter, and of a like nature to the earth, would attract a small body placed near its surface with a force 20,000,000 times less than the earth would do if placed near its surface; but so small a force could produce no sensible effect. If two such spheres were distant but by ¼ of an inch, they would not, even in spaces void of resistance, come together by the force of their mutual attraction in less than a month's time; and less spheres will come together at a rate yet slower—viz., in the proportion of their diameters. Nay, whole mountains will not be sufficient to produce any sensible effect. A mountain of a hemispherical figure, three miles high and six broad, will not, by its attraction, draw the pendulum two minutes out of the true perpendicular" (Davis's edition of Motte's translation of the "Principia," etc., 1803, vol. iii. p. 22). Unfortunately, there are two errors in the earlier part of the quotation, one of them leading to an enormous under-estimate of the effect of the attraction of spheres on each other. In the first place, the diameter of the earth is about 40,000,000 feet, so that the pull of two spheres on each other, one foot diameter each, one foot from centre to centre, and of the earth's mean density, would be $\frac{1}{4} \times \frac{1}{40000000}$ of the weight of either. Secondly, the acceleration towards each other would be this fraction of 32 feet / sec.^{2} or of 384 inches / sec.^{2}, and the time taken to move $\frac{1}{8}$ inch would be given by $\frac{1}{8} = \frac{1}{2}\, t^2 \times \frac{384}{160000000}$, whence $t = 320$ seconds, about, instead of a month. Had Newton realised this, he would hardly have dismissed so summarily the possibility of observing the attraction. Still, the passage deserves attention as showing that Newton fully recognised the two methods of experiment by which any effect should be sought for, the only two methods which have yet been discovered.

The Experiments of Bouguer in Peru.

(No. 3 in the Bibliography.)

The honour of making the first experiments on the attraction of terrestrial masses is to be accorded to Bouguer, a member of the celebrated commission sent in 1735 by the French Academy of Sciences to measure an arc of the meridian in Peru. He attempted, both by pendulum and by plumb-line experiments, to prove the existence, and to measure the attraction, of mountain masses, and the seventh section of his "Figure de la Terre," entitled "Détail des Expériences ou Observations sur la Gravitation, avec des remarques sur les causes de la Figure de la Terre," contains a full account of the work. These experiments cannot be regarded as having any quantitative value. But, as being the first made, as pointing out the methods, and as certainly showing the existence of the attraction sought, they are important in the history of the subject. Regarding them as preliminary trials, I shall give an account of the plumb-line and pendulum experiments together, instead of dealing with them under the separate classes with the later experiments.

The Pendulum Experiment.—Bouguer determined the length of the seconds pendulum at three elevations: (1) at Quito, which may be regarded as on a table-land, the station being 1466 toises above sea-level; (2) on the summit of Pichincha, a mountain rising above Quito to a height of 2434 toises above sea-level; and (3) on the island of Inca in the river Esmeralda, not more than 30 or 40 toises above sea-level. This station is to the north-west of

Quito, and about half a degree north of that town in latitude, the equator lying between them. The toise being taken as 6·395 feet, we may put the elevations in descending order as 15,600 feet, 9,400 feet, and 0. The Quito observations were made at various times: those on Pichincha in 1737, and those at the Isle of Inca in 1740.

The pendulum consisted of a small double truncated cone of copper suspended from fixed jaws by an aloe fibre. The thread was always made of the same length as a certain iron measuring rod, so that the pendulum was "invariable." The time of vibration was found from the number of oscillations gained in a given time on a seconds clock rated by astronomical observations. The temperature of Quito was regarded as equal to that of Paris in the middle of spring, and the measuring rod was taken as correct there. It was therefore necessary to apply temperature corrections to the lengths obtained at the higher and lower levels. Though thermometers were used, the corrections do not appear to have been more than approximate estimates. The buoyancy of the air was also allowed for, its value on Pichincha being determined as $\frac{1}{11000}$ of the weight of the copper bob by noticing how far the barometer must be carried down hill to rise one line.

The following table gives the results:

Station.	Above Sea Level. In Toises.	Observed Seconds Pendulum in Lines.	Correction for Temperature.	Correction for Buoyancy.	Corrected Seconds Pendulum.	Fraction Less than at Sea Level.	Fraction Given by Inverse Square Law $2h/r$.
Pichincha	2,434	438·70	−·05	+·04	438·69	$\frac{1}{845}$	$\frac{1}{674}$
Quito	1,466	438·83	—	+·05	438·88	$\frac{1}{1331}$	$\frac{1}{1118}$
Isle of Inca	—	439·07	+·075	+·06	439·21		

In a matter-free space rising above sea-level, gravity might be expected to decrease according to the inverse square law starting from the earth's centre, so that if h is the height above sea-level and r is the earth's radius the decrease should be $2h/r$. The value of this fraction is given in the last column, and on comparing it with the observed decrease in the preceding column it will be evident that gravity decreased more slowly than by the inverse square law. Centrifugal force would act in a contrary way, though, as Bouguer showed, by a negligible amount. The excess of gravity, as observed, above its value in a free space must therefore be assigned to the attraction of the matter above the sea-level.

Bouguer obtained for the value of gravity on a plateau of height h as compared with its value at sea level

$$g_1 = g\left(1 - \frac{2h}{r} + \frac{3h}{r}\frac{d}{D}\right)$$

where d is the density of the plateau and D that of the earth.

This formula seems to have dropped out of sight till it was revived by Young in 1819.

Putting it in the form

$$\frac{g - g_1}{g} = \frac{2h}{r}\left(1 - \frac{3}{2}\frac{d}{\mathrm{D}}\right)$$

and using the values at Quito and sea-level—

$$\mathrm{D} = \frac{3993}{850} d.$$

This result, though we now know it to be far too great, at least sufficed to show that the mean density of the earth was greater than that of the Cordilleras, and consequently, as Bouguer remarked, that the interior of the Earth was neither hollow nor full of water, as some physicists had maintained.

The Plumb-line Experiment.—After the account of the pendulum experiments Bouguer proceeds to discuss the methods by which the deflection of the plumb-line through mountain attraction may be measured, and he then gives the account of his own observations on Chimborazo. He points out that several methods are available, *e.g.*, the meridian altitude of a star may be observed by a quadrant with its zenith reading fixed by a plumb-line, at a point on a mountain side either to the north or to the south of the summit, and then at a point east or west of the first station out of the range of the mountain's attraction. The difference should give the deflection of the plumb-line due to the mountain. This is the method which he used himself. Or, selecting stations on the same meridian, respectively on the north and south slopes, the excess of the difference of astronomical latitude over the geodetic latitude is twice the deflection due to the mountain. This is the method afterwards used by Maskelyne and by James and Clarke. Or the stations may be selected one on the north side of one mountain, and the other on the south side of another more northerly mountain. Or, lastly, the stations may be selected on the east and west sides of a mountain. This method has never been used, but it appears worthy of notice for its ingenuity. I therefore give it in Bouguer's own words ("Figure de la Terre," p. 372, § 60):

"Enfin ce n'est pas seulement par des observations faites au Nord ou au Sud, qu'on peut découvrir si les montagnes sont capables d'agir *en distance ;* c'est aussi par des observations faites à l'Orient ou à l'Occident ; avec cette seule différence qu'il ne sera plus question d'observer la latitude, ou de prendre des hauteurs méridiennes d'Etoiles et qu'il ne s'agira que de la détermination exacte de l'heure. Cette dernière méthode qui peut avoir son utilité, me paroîtroit souvent préférable aux précédentes, sans qu'elle exige toujours le concours de deux Observateurs. Supposons que le premier soit situé sur le côté Occidental d'une autre montagne ou de la même: si chacun de ces Observateurs prend soin de regler une pendule par des hauteurs correspondantes, il est évident que toutes ces hauteurs étant alterées par l'attraction que souffre la ligne verticale, chaque pendule sera reglée comme si le Méridien n'etoit pas exactement vertical et comme s'il s'étoit approché par en bas de la montagne, et éloigné par consequent par en haut. Supposé outre cela que l'attraction soit d'une minute de degré et que les deux montagnes soient sur l'Equateur, la première Horloge marquera le midi 4 secondes de tems trop tôt et l'autre 4 secondes trop tard. Ainsi faisant abstraction de la différence des longitudes qu'on découvrira aisément en mesurant par la Trigonométrie la distance des deux Observateurs et en l'evaluant en degrés et minutes, il y auroit 8 secondes de tems de différence entre les deux pendules. Si les deux montagnes au lieu d'être placées aux environs de l'Equateur etoient par 60 degrés de latitude, chaque minute d'inclinaison que produiroit l'attraction dans la ligne verticale apparente, peut apporter 8″ de différence dans le midi, et il y en auroit donc 16 entre les deux Horloges. Enfin pour juger de la quantité de l'attraction, il n'y auroit qu'à sçavoir exactement la différence qu'il y auroit entre les deux Horloges; et il suffiroit toujours, pour la découvrir, de convenir d'un signal de feu ou de quelqu'autre signal dont on pût saisir l'instant; et remarquer de part et d'autre la minute et la seconde de son apparition."

After considering several mountains, Bouguer fixed upon Chimborazo and selected as his first station a point on the south side, nearly in the same longitude as the centre of gravity, and just about the snow-line. The station was 2400 toises above sea level and 829 toises below the summit. After a most toilsome journey, Bouguer and de la Condamine, another member of the Commission who assisted him in this work, fixed their tent at the station on December 4, 1738. Their difficulties were greatly increased by the cold, and they had continually to guard against being buried by the snow. They succeeded, however, in rating their clock, and then they proceeded to take the meridian altitudes of six stars to the north and four to the south, by means of a quadrant of $2\frac{1}{2}$ feet radius. On December 16th they moved to the second station nearly to the west of the first, about 3500 toises distant, at a level 174 toises lower, and 32 seconds more south in latitude. Here the attraction of the mountain in the northerly direction was only about $\frac{1}{13}$ or $\frac{1}{14}$ of its value at the first station. Though at a lower level their condition was worse than before, for they had changed a sheltered place for one exposed to such a wind that, as Bouguer says, "il nous remplissoit les yeux de sable et il étoit continuellement sur le point d'enlever notre tentre." Nevertheless they succeeded in observing the altitudes of the same stars as before.

After applying corrections for the geodetic difference of latitude and the difference of refractions at the two stations, the results were treated thus: Let N be the true altitude of a north star, δ the plumb-line deflection, and z an error in the reading of the quadrant. Then at the first station the observed altitude was,

$$N - \delta + z \qquad (1)$$

If on the same night a south star was observed, z might be taken as the same and the observed altitude was:

$$S + \delta + z \qquad (2)$$

At the second station the observations being made at a different time, we must assume a different error z'. After correction for latitude and refraction the observed quantities are:

$$N + z' \qquad (3)$$

$$S + z' \qquad (4)$$

Then $$(3) - (1) - \{(4) - (2)\} = 2\delta.$$

The mean of very discordant values of 2δ was 15″. Correcting δ for the residual attraction, $\frac{1}{14}$th, at the second station, we may take the value of δ as 8″.

From an estimate of the volume of matter above the stations and its distance, it was calculated that with the same density as the earth it should produce a deflection of 1′ 43″.

Bouguer seemed to think that the smallness of the result might be due to light volcanic matter in the mountain, but he recognised fully the unsatisfactory nature of the work, and does not seem to have had any confidence in the result. He expressed the belief that some suitable place might be found in France or in England for a repetition of the experiment, and hoped that on his return to Europe he should find that means had been taken to throw further light on the subject.

Though this hope was unfulfilled and his experiments had no immediate effect, interest in the subject was awakened, and Bouguer's work undoubtedly inspired Maskelyne to make the celebrated Schehallien experiment now to be described.

PLUMB-LINE EXPERIMENTS.

The Schehallien Experiment by Maskelyne and Hutton.

(*Nos.* 4, 5, 6, 7, 9 & 12, *in the Bibliography.*)

In 1772 the Rev. Nevil Maskelyne, Astronomer Royal, read a paper at the Royal Society, in which he proposed a repetition of the attempt to measure the attraction of a mountain in the manner adopted by the Peruvian Commission. He suggested that a suitable locality might be found either in the Ingleborough district or in Cumberland, and obtained the appointment of a committee by the Society "to consider of a proper hill whereon to try the experiment, and to prepare everything necessary for carrying the design into execution."

As the result of the inquiries and inspection made by this Committee, the English hills were rejected in favour of Schehallien, a mountain near Rannoch, in Perthshire, which has a short ridge nearly east and west, sloping down steeply on the north and south. The summit is, according to the Ordnance Survey since made, 3547 feet above sea-level.

In the summer of 1774 the necessary instruments and a portable observatory were despatched thither. Two stations, the north about 1450 and the south about 1180 feet below the ridge and on the same meridian, were selected and the astronomical observations were made. These consisted in the determination at each of the two stations of the meridian zenith distances of a certain number of stars passing the meridian near the zenith. A survey of the hill and of the surrounding district was begun in 1774, and completed during the two following years. This gave the geodetical or geographical difference of latitude of the two stations, and, had the mountain not existed, or had its attraction been negligible, the difference of zenith distances of the same star as observed from the two stations, or its equivalent, the angle between the plumb-lines at the two, would have been the same as this geodetical difference. But the attraction of the mountain pulled the plumb-bob in opposite directions on the two sides, and so increased the astronomical difference, and this increase was the quantity sought.

Knowing, from the survey, the disposition and distance of the disturbing attracting matter, it was possible by calculation to determine the mass of the earth in terms of that of the disturbing matter.

For the determination of the zenith distances, Maskelyne used a zenith sector, an instrument designed only for stars very near the zenith. This consisted of a 10-foot telescope turning on a horizontal axis at the objective end, and provided with a divided arc of a few degrees at the eye-piece end. A plumb-line, passing through the horizontal axis hung down against the divided arc and fixed the zenith. The eye-piece was provided with a micrometer. It is, perhaps, worthy of note that at that time there was no satisfactory method of dividing an arc to degrees, and Maskelyne graduated his arc by laying off a chord equal to $\frac{1}{8}$ radius and then by continued bisection dividing its arc into 128 equal parts. In using the instrument the nearest division was always brought exactly under the plumb-line, and fractions of a division were determined by a micrometer. The sector was provided with a vertical axis and was fixed in the meridian by the aid of observations made with a quadrant, but owing to delays in effecting this through bad weather, Maskelyne was led to devise a plan of determining the meridian which is now commonplace. This consists in observing the times of transit of two known stars and adjusting the instrument till the difference coincides with the known difference of right ascensions.

At the south station, 76 observations were first made on 34 stars, and then, turning the instrument through 180° round its vertical axis, 93 observations on 39, chiefly the same stars. At the north station 68 observations on 32 stars were made with the face of the instrument west, and 100 on 37 with the face east.

After making due allowance for "precession, aberration and deviation and semi-annual solar nutation of the earth's axis," ten differences with the face of the sector west gave a mean of 54·2″, the greatest deviation being 2″. Ten differences with the face east gave a mean of 55″, the greatest deviation again being 2″. The mean of these 54·6″ was taken as the final result.

Meanwhile, the survey gave the difference of geographical latitude as 42.94″.

Whence the effect due to attraction of the disturbing matter was 11·6″.

So far, the work is Maskelyne's, but the computation of the result from these data was carried out in a most careful and ingenious way by Hutton. His plan was as follows:

From the data of the survey, he drew a contour map of the district, and, from his description, it would appear that he was the originator of this mode of representing the surface.

He then supposed a horizontal plane to be drawn through each station.

Taking one, say the south station, had the surface of the ground been everywhere level with this plane, the plumb-line would have coincided with the geographical normal. The actual deviation towards the north might be set down as due to the horizontal northerly component of the attraction of the matter above the plane, the defect of matter where the surface fell below being considered as negative matter above it. The attraction was then calculated on the supposition that unit volume at unit distance has unit attraction, and to simplify the calculation, the matter was divided into vertical prisms by vertical cylinders, with radii increasing successively by equal amounts of $666\frac{2}{3}$ feet from the station, and vertical radial planes at angles with the meridian, such that there was a constant difference $\frac{1}{12}$ between the successive sines. It is easily shown that the northerly horizontal component of the attraction of a ring between two cylinders is $\frac{666\cdot\dot{6}}{12}$ × sum of $h/\,(r^2 + h^2)^{\frac{1}{2}}$ where h is the height of a prism above the horizontal plane. An ingenious mode of computing this from the sum of h/r, by the aid of a slide rule, was devised by Cavendish, who was also partly responsible for the method of dividing up the ground into prisms. He was evidently already interested in the problem which he was himself to attack in another way twenty years later.

Hutton used, in all, twenty rings round each observatory, extending to 13,000 feet away, and the results obtained were,

$$\text{For the south station } \frac{666\cdot\dot{6}}{12} \times 69\cdot967$$

$$\text{,, north ,, } \frac{666\cdot\dot{6}}{12} \times 88\cdot644,$$

the sum of these being nearly $8811\frac{2}{3}$ the attraction of the whole earth would in the same units be $4\,\pi\,R/3$ or $\frac{2}{3}$ circumference.

The circumference was taken as 131284080 feet, a value, to one in three thousand, the same as that used by Clarke eighty years later.

Hence on the supposition of equal mean densities:

$$\frac{\text{Sum of attractions of hill}}{\text{attraction of earth}} = \frac{8811\cdot\dot{6}}{8752270} = \frac{1}{9933}$$

But Maskelyne found the ratio of the two equal to the circular measure of 11·6″ or $\frac{1}{17781}$; or $\frac{1}{17804}$ after allowing for "Centrifugal force."

$$\text{Hence,} \qquad \frac{\text{density of earth}}{\text{density of hill}} = \frac{17804}{9933} = \frac{9}{5} \text{ say.}$$

Hutton took the specific gravity of the hill as 2·5, whence the mean density of the earth comes out 4·5 times that of water.

Many years later, at Hutton's instigation, Playfair undertook a lithological survey of the hill. He found that the strata were nearly vertical, and he observed the position and quantity of each kind of rock. He also took specimens of each and determined their specific gravities. He was thus enabled to apply corrections to Hutton's calculations, and gave, as the limits between which the result lay, 4·5588 and 4·867 (*Phil. Trans.*, 1811).

Hutton's latest contribution to the subject (*Phil. Trans.*, 1821, pp. 276–292), contains his final estimate (based on Playfair's work) of 99/20 or 4·95, a result which he had previously published in his Tracts. There is also a noteworthy suggestion that one of the large pyramids in Egypt, might be employed instead of a mountain, the stations being selected a quarter of the way up from the base, the position which he had shown (*Phil. Trans.*, 1780, p. 1) to give the largest deflection. The rest of the paper is occupied with a résumé of the Schehallien experiment, and a criticism of Cavendish's arithmetic, a criticism afterwards shown by Baily to be itself erroneous.

It should be explained that Hutton was now eighty-four years old, and broken down in health. In his earlier years he was too exact and careful to have made any such criticism. He appears to have felt that Cavendish's value was being accepted in preference to his own and Maskelyne's, without due reason, and doubtless this made him more ready to discover what he fancied to be errors.

Reviewing the Schehallien experiment it is easy now to point out weak points. In the first place a larger number of astronomical observations with a closer agreement would have been more satisfactory. Then, in the second place, the calculations hardly extended far enough from the hill. The only justification in stopping at any distance is that the matter beyond would produce practically the same effect at the two stations. On examining Hutton's tables, I find that the last rings retained produce far too great a fraction of the whole difference to justify the neglect of those beyond, and it would be necessary to go some distance further, especially as from the map it would appear that the ground rises to the north some 20,000 feet away, and still within a distance giving a differential effect. And thirdly, the determinations of the specific gravity of each kind of rock, and its exact quantity and position, should all receive as much attention as the astronomical and survey part of the work. But this is practically impossible, and constitutes a weakness inherent in the method.

Notwithstanding these weak points the experiment was a great attempt to solve the problem, and considering all the labours and the difficulties of the work, much greater then than they would be now, we must recognise that it was most admirably carried out, and that it fully deserves the high place which it occupies in the history of scientific achievements.

The Survey Experiment at Arthur's Seat by Colonel James and Captain Clarke.

(*No.* 24 *in the Bibliography.*)

On the completion of the Trigonometrical Survey of the United Kingdom, it was found that there were very sensible differences between the astronomical and geodetic latitudes at several places, and notably at Edinburgh, where there was more than 5″. The difference could usually be ascribed to the local configuration, and in order to confirm this explanation Col. James, the Superintendent of the Survey, in 1855, decided to examine the Edinburgh case more thoroughly by determining the difference of latitude on the two sides of Arthur's Seat astronomically, and comparing it with the geodetical difference given by the Survey.

A mineralogical survey of the surface strata of Arthur's Seat and the surrounding district was made by Col. James himself, the astronomical observations were made by Sergeant-Major Steel and the subsequent computations based on the contoured maps already prepared were undertaken by Captain Clarke.

The astronomical part of the work consisted in the determination of the zenith distances of a number of stars by over 400 observations at each of three stations N, A, and S, respectively on the north side, the summit, and the south side.

The zenith sector of Airy was used, an instrument which may be regarded as the modern equivalent of Maskelyne's zenith sector, a level being used however instead of a plumb-line. In Clarke's "Geodesy," pp. 182–4, will be found a description of the instrument.

The method of least squares was applied to the observations to extract the most probable values of the differences of astronomical latitudes. The latitude of the south station given by the stars observed there was 55° 56′ 26″·69, and assuming this to be correct the results in the second column of the table below were obtained, the third column giving the results of the Survey and the last the differences:

Station.	Astronomical Latitude.	Geodetical Latitude.	Difference. A – G.
S	55° 56′ 26·69″	55° 56′ 24·25″	2·44″
A	55 56 43·69	55 56 38·44	5·25
N	55 57 9·22	55 57 2·71	6·51

Evidently, then, there is a deflection of the plumb-line to the north, or an attraction towards the south, at all three stations. This is greater even than the northerly deflection due to Arthur's Seat at the south station, a notable result to which we shall return.

To calculate the attraction due to the observed configuration Clarke used the method of Hutton, somewhat modified for greater exactness. He drew 5 circles at 100 feet intervals round each centre, then 15 more at 500 feet intervals round N and S, and 11 more at the same intervals round A. Then 7 more round N and S, and nine more round A, each having $\frac{7}{8}$ the radius of the preceding. This took him out nearly 24,000 feet. Dividing each quadrant into 12 sectors by Hutton's plan, and computing the attraction he obtained an equation for each station. If we put x for density of rocks / density of earth, and y for the deflection due to matter beyond the last rings retained, the following are the equations, the meaning of each term being written above:

A − G		Deflection of Outside Matter.	Deflection of First Twelve or Sixteen Rings.	Deflection of Outer Rings.
2·44	=	y −	4·265 x	+ 1·565 x
5·25	=	y	+ 1·691 x	+ 0·708 x
6·51	=	y	+ 3·845 x	1·393 x

Whence x = ·5173; y = 3·8820.

If the surface strata have density 2·75, the value assigned by Col. James, then:

$$\text{Mean density of earth} = 2{\cdot}75 / {\cdot}5173 = 5{\cdot}316.$$

Stopping short of the outer rings, *i.e.*, omitting the last term in each equation, the value of x = ·5076, and

$$\text{Mean density} = 5{\cdot}417,$$

a difference showing the importance of the outer rings and justifying the remark that Hutton did not go far enough.

The work on which the above result is based appears to have been carried out most thoroughly and carefully. The astronomical observations agreed so closely that the probable error due to them is only ± ·054.

The calculation of the attraction of the surrounding irregularities was carried probably quite as far as was necessary when we consider our ignorance of the underlying material and the only source of considerable uncertainty is one not to be eliminated from this method—viz., the value assigned to the density of the deflecting matter, a value which may easily be wrong by as much as 5 per cent.

The General Deflection at all three Stations.—The question of the origin of the term y, the deflection due to outside matter, has no very direct bearing on our subject, but it is of great importance in the allied subject of the deflection of the plumb-line generally by table lands and mountain ranges, and the accompanying alteration in gravity as shown by the pendulum. A short account of Col. Clarke's calculations in his search for an explanation may therefore not be out of place.

To the north of Edinburgh there is a defect of matter due to the Forth.

The hollow of this would be approximately covered by a rectangle 18 miles long, 12 miles wide, with its shorter side at 50° to the meridian, and having Arthur's Seat two miles away opposite its middle point. A plate of this size, depth h feet and density x as compared with that of the earth, would produce deflection $\cdot 00727\, x\, h$.

Taking $h=30$—the mean depth of the Forth—and $x=1/5$, the effect of the water is to produce a deflection of ·04″ to the north-east. Taking fifteen feet as the difference between high and low water, there may be a variation of ·02″ with the state of the tide. Removing the water and filling up the rectangle with rock of specific gravity 2·5 to a depth of 150 feet, we should get a deflection greater than that due to the water by 0·5″. Hence the hollow of the Forth and the consequent northerly defect of matter will account for very little of the deflection of nearly 4″—or 5″, if we accept the results of the general Survey.

Turning now to the south, it is to be observed that the surface gradually rises to a mean level at the south of the country of about 1000 feet. Clarke extended the circles from the 21st to the 28th, each having $\frac{7}{6}$ the radius of the last, and this brought him to the borders of Edinburghshire, some thirteen miles away. The attraction due to the extra matter thus included was found to be 1·64″.

Further extension could not be made owing to the want of contoured maps, but Colonel Clarke thought that if it could be made, possibly the whole deflection might be accounted for.

It may be pointed out that, on the plan adopted for drawing the rings, the attraction of any ring depends only on its average height and not on its distance, so that an addition of ten more rings, the last about 60 miles radius and all of the average height of the preceding seven, would produce a deflection of $1{\cdot}64 \times \frac{10}{7} = 2{\cdot}34''$. Adding this to the 1·64″ already found, we have a total of 3·98″, or more than Clarke's value of the deflection. But this would assume that all the country south has the high average level of the borders of the counties of Edinburgh and Peebles, and would leave out of account the highlands of Perthshire now just coming within the radius on the other side. And here we are brought face to face with the question, Where is this process to end? An addition of ten rings means a multiplication of radius by 4·67, say roughly 5. Suppose we added another ten, the last with a radius of 300 miles, we should include on one side the greater part of England and Wales, and on the other, all the Highlands; and this ten is worthy of as much consideration as either of the preceding groups of ten. Indeed, it becomes evident that we can have no certainty whatever in our explanation of any one case of deflection, taken by itself, since distant masses by their greater area may become as important as the nearer ones which, at first sight, we might regard as alone worthy of consideration. And, indeed, the process of calculation is apparently faulty in another respect. Everything tends to show that elevated ground of great extent, such as a group of mountains or a table-land, never exerts the attraction we should expect from its observed density and it is hard to resist the supposition first made by Airy (*Phil. Trans.*, 1855, pp. 101–4), that these elevated masses are really buoyed up by matter at their base, lighter than the average; that, in fact, they float on the liquid, or more probably viscous solid, interior very much as icebergs float on the sea. If this high ground is in equilibrium, neither rising nor falling, we may perhaps regard the total quantity of matter as the same as if all were uniform and at the sea level. Then the excess of horizontal attraction on a distant point, due to the elevated matter, would be neutralised by the defect due to the lighter matter below, and the elevations might be left out of the account.

But, perhaps, with sinking areas the matter below is not sufficiently buoyant and there might be an excess of attraction, while with rising areas the matter below may be more than sufficiently buoyant, and there may be a defect of attraction. In our present want of knowledge on these points it appears useless to take distant matter into account in order to explain any special case of difference between astronomic and geodetic latitude. The direction in which there appears some hope of success is rather in the experimental investigation of the deflection of the plumb-line at many points over large areas and the determination of such a distribution of excess or defect of matter as would account for all these differences.

Then we should be able to decide as to the truth of Airy's flotation theory, or at least we might find whether it was possible.

PENDULUM EXPERIMENTS.

Carlini's Experiment on Mont Cenis.

(*Nos.* 13 *and* 18 *in the Bibliography.*)

In 1821, Carlini and Plana were engaged in determining the difference of longitude between the Hospice on Mont Cenis and the Observatory at Milan by observing signals made at an intermediate point. Carlini was at the Mont Cenis end, and having very accurate time for the purpose of comparison with Milan time, he made use of his stay at the Hospice to determine the value of gravity there by a Borda's pendulum. This consisted of a platinum disc suspended by a metallic thread from a "rotella" weighing only ten grains, instead of the ordinary knife edge. The length of the pendulum was measured when in position in its case by means of two microscopes, the upper sighting the suspension edge, the lower, in succession, the upper and lower edges of the disc. The length between the microscopes was measured in terms of three agreeing standard metres. The pendulum and microscope were attached to a strong wall on the ground floor, and the clock was on a pyramid of masonry rising in the middle of the room. A telescope was directed to the simple pendulum, and an inclined mirror at the same time reflected into the telescope the image of the clock pendulum so that their coincidences could be observed.

The experiments extended over twenty-five days with an interval, and thirteen independent results of length and time of swing were obtained. The mean result, when corrected from the elevation of the Hospice (1943 metres) to sea level, on the supposition that no mountain existed, was 993·708 mm., with a maximum discordance of ·032 mm. The excess due to the mountain was thus brought down, as it were, to sea level.

Biot had found 741·6151 mm. as the length of the "decimal" seconds pendulum at Bordeaux, lat. 44° 50′ 25″. Correcting to the latitude of the Hospice 45° 14′ 10″, and expressing in "sexagesimal" seconds the length of the pendulum there at sea-level should be 993·498 mm. The excess of the observed result 0·210 mm. was taken as the effect due to the mountain.

Carlini considered that the mountain might be represented by a segment of a sphere 1 geographical mile high and with a base 11 miles in diameter. The attraction of such an elevation works out to ·5020 *d*, where *d* is the density of the mountain and 1 mile is the unit of length. The attraction of the whole earth in the same units is 14394 D, where D is the mean density of the earth, and these attractions should be in the ratio of the difference 0·210 mm. to the whole length of the seconds-pendulum 993·498 mm.

The mountain consists chiefly of schist, marble, and gypsum, and Carlini obtained the specific gravities of these from a number of specimens, finding them to be for schist 2·81, for marble 2·86, and for gypsum 2·32. He took the mean 2·66 as approximately the value of *d*.

$$\text{Whence} \quad D = \frac{5{\cdot}02 \times 993498 \times 2{\cdot}66}{14394 \times 210} = 4{\cdot}39$$

Sabine (Bibliography No. 11) gives a different treatment of the observations, leading to a different result. According to Sabine the actual length of the seconds pendulum at the Hospice is 39·0992 in. From five values taken in neighbouring latitudes at Dunkirk, Paris, Bordeaux, Figeac, and Formentara, all nearly at sea-level, the sea-level value in the latitude of Mt. Cenis found by interpolation is 39·1154 in. The difference between this and the Hospice length is ·0162 in. But were the mountain cleared away the difference due to the altitude of the Hospice should be ·0238. The effect of the mountain is therefore the excess of ·0238 over ·0162 = ·0076.

$$\text{Hence } D = \frac{5{\cdot}02 \times 39{\cdot}1154 \times 2{\cdot}66}{14394 \times {\cdot}0076} = 4{\cdot}77$$

A subsequent revision by Giulio (Bibl. No. 18) led to the value 4·95.

There are very obvious weak points in this work. The variation from the mean of ·032 in a total of ·210, in only thirteen observations is somewhat large and would render the result too uncertain, even if the value at the sea-level as obtained by interpolation could be trusted. But there is still more uncertainty in the estimate of the equivalent mountain, and in the value assumed for the specific gravity of the rocks. The experiment must, I think, be regarded as interesting rather than important, and the nearness of the value to that obtained by other methods is probably accidental, and not an evidence of the exactness of the data and the assumptions.

Airy's Experiment at the Harton Pit.

(*Nos.* 14, 15, 16, 22 *and* 23 *in the Bibliography*).

In the year 1826, the idea occurred to Mr. Airy (afterwards Sir George Airy and Astronomer Royal) that the mean density of the earth might be determined by observing the difference in the rate of a pendulum at the top and bottom of a mine. Assuming the earth to consist of homogeneous concentric spherical shells, let r be the radius of the sphere through the bottom of the mine, $r + h$ that of the outer surface, D the mean density of the inner sphere, d that of the shell penetrated by the mine. Let g be the value of gravity below, and g' that above.

$$\text{Then } g / g' = \frac{4}{3} \pi r D \Big/ \left\{ \frac{4}{3} \pi r^3 D / (r + h)^2 + 4 \pi h d \right\}$$

$$= 1 + \frac{2h}{r} - \frac{3h}{r} \frac{d}{D} \text{ nearly,}$$

$$\text{and} \quad \frac{D}{d} = \frac{3h}{r} \Big/ \left(1 + \frac{2h}{r} - \frac{g}{g'}\right)$$

To find the error in D due to an error in g, differentiate, and it is easily shown that:

$$\frac{\delta D}{D} = \frac{\delta g}{g} \cdot \frac{D}{d} \cdot \frac{r}{3h}.$$

Putting $D = 2d$, $h = \frac{1}{4}\ m$, $r = 4000\ m$, any error in g' is multiplied roughly by 10^4 in D.

Hence, to get within 1 per cent. in D, we must find g / g' to one in a million. This appeared possible to Airy, and in conjunction with Whewell he made plans to undertake a series of observations on the relative values of gravity at the top and bottom of a copper mine 1200 feet deep, at Dolcoath, near Camborne, in Cornwall. Two invariable pendulums were to be compared with two clocks, respectively at the top and bottom of the mine, the clocks being compared by chronometers carried to and fro. The pendulums were then to be interchanged, and any difference in their rates with the same value of gravity would thus be eliminated. After some trouble with the chronometer method of comparing the clocks, one set of observations of rate of swing was made. In the process of interchange, one of the pendulum boxes took fire in some unknown way while being raised in the shaft. It fell to the bottom and the accident terminated the experiment.

In 1828, a repetition was attempted in the same mine, Mr. Sheepshanks joining the party. The irregularity of the knife edges gave great trouble, and when the difficulty seemed to be overcome, a fall occurred in the mine which stopped the pump and the lower station was flooded, so that again the experiment had to be abandoned.

Airy published no account of the result of these first attempts, but from Drobisch's first paper it would appear that the difference of rate of the two pendulums corresponded to a mean density of 20, clearly showing that the experiment was still in the preliminary state.

No further steps were taken for many years.

Meanwhile, the method of electric signalling had come into use and rendered the comparison of distant clocks an easy matter, so that, when in 1854 Airy once more returned to the subject, one part of the experiment was simplified, and he had for the rest all the facilities afforded by his position as Astronomer Royal.

He now selected the Harton Pit, a coal mine in the county of Durham, two miles south of South Shields. An upper station was chosen in a stable about 200 yards from the mouth of the shaft, and a lower one in the same vertical in a disused gallery some 1250 feet below. An inner chamber 16 feet square was built in the stable and a similar chamber in the gallery as clock and pendulum rooms.

The plan was essentially that of the earlier attempts. Two invariable pendulums, belonging to the Royal Society, marked respectively 1821 and No. 8, were used. At each station was a clock, in front of which one of the pendulums was hung, and their rates were compared by means of the method of coincidences.

To compare the two clocks a "journeyman" clock was fitted up with an electric circuit in the upper station, the circuit passing down the shaft to the lower station. Every 15 seconds the circuit was completed, and a galvanometer placed at the side of each comparison clock was deflected. By preconcerted signals it was known which deflections of the galvanometers were to be timed on the two clocks, so that their indications at the same set of instants were known, and, by successive comparisons at definite intervals their rates could be compared.

The invariable pendulum stands may be described as each consisting of two iron triangular frames with a common side—one triangle horizontal, the other vertical. A bar connecting their vertices strengthened the frame-work. At the vertex of the vertical triangle was fixed the support for the agate planes on which the steel knife-edges of the pendulum rested. The base rested on three bricks, with perfectly solid bearings, one at each angle, and the comparison clock was placed within the frame-work but free from it. The two stands were arranged so that the pendulums vibrated magnetic east and west.

There were in all six observers, and arrangements were made by which a continuous series of swings should be observed, night and day, each swing occupying about four hours. Some four or five coincidences were observed at the beginning and end of each swing, and the arc of vibration was also noted at the beginning and end. Between each pair of swings the amplitude was increased to its original value, and comparisons of the upper and lower clocks were made. Temperature and pressure observations were also made for the purposes of correction. In all, 82 swings were observed, divided into four groups, an interchange of pendulums taking place when each group was completed. The observer at a given station was relieved every twelve hours, so that the clock comparisons were sometimes made by different observers and personal equation came in. By the method of rotation of observers at the two stations, the effect was practically almost eliminated, but Airy considered it better to determine the personal equation of each observer from the actual comparisons and allow for it in the results.

It was originally intended to keep the two stations always at the same temperature, but through an accident a difference was allowed to exist, the lower station being on the average almost 7° F. warmer than the upper. The assumed temperature correction, if erroneous, might therefore seriously affect the result. A series of comparisons of the pendulums at Greenwich at the conclusion of the Harton Pit experiment made at very different temperatures, showed, however, that the correction was practically right.

When all the corrections for a pendulum have been made its rate, or the number of vibrations per second, should be proportional to the square root of g. Hence we have:

$$\frac{g \text{ above}}{g \text{ below}} = \frac{(\text{rate of a pendulum above})^2}{(\text{rate of same pendulum below})^2}.$$

Denoting the pendulums used by "8" and "1821," this may be arranged as follows:

$$\begin{aligned} &= \frac{\text{rate of ``8'' above}}{\text{rate of upper clock}} \times \frac{\text{rate of ``1821'' above}}{\text{rate of upper clock}} \\ &\times \frac{\text{rate of lower clock}}{\text{rate of ``8'' below}} \times \frac{\text{rate of lower clock}}{\text{rate of 1821 below}} \\ &\times \left(\frac{\text{rate of upper clock}}{\text{rate of lower clock}}\right)^2 \end{aligned}$$

for $\dfrac{8 \text{ above}}{8 \text{ below}} = \dfrac{1821 \text{ above}}{1821 \text{ below}}$, if the corrections have been truly applied. But here all the quantities are directly observed and so the ratio of gravity above to gravity below is determined.

When all the separate observations were weighted so as to give the least probable error, it was found that gravity below exceeded gravity above by $\frac{1}{19286}$, with a probable error of 1 in 270.

The next step is to express the difference of vertical attraction at the two stations, in terms of the whole earth, and of the strata surrounding the pit.

Let us first suppose the hollows in the upper surface filled up with material of the density of the surrounding strata, and the elevations removed so that the surface is truly spherical, and let us assume that the stratification is level.

Then, drawing a shell through the lower station such that the matter without it has no attraction at that station, the form of this shell will be spherical everywhere near the station, whatever may be its distant irregularities, and it is easy to see that its attraction on the upper station will be $4 \pi h d$, where h is the depth of the mine, and d the average density of the surface strata. The material within the shell may be regarded as concentrated at the centre of the earth for both stations, and so we may apply the spherical formula, given at the beginning of this account. Now, restoring the surface to its true form a correction has to be made for the difference of attractions of the removed or replaced matter at the two stations.

The ground round the Harton Pit is a table-land with only small irregularities, extending to a line of cliffs at the coast, about 70 feet high, and 12,000 feet from the mine at the nearest point.

The ground irregularities might be considered as neutralising each other at the upper station, and at the lower their effect was computed by dividing the surface into squares by lines parallel and perpendicular to the coast, each of side length h. The average elevation or depression above or below the upper station, was determined for each square by a survey, and so the effect at the station below could be computed.

For the sea, the surface was taken as fifteen feet below high water, and its effect had to be calculated for both stations.

The result was that the attraction of the shell on the upper station, in terms of the depth of the mine as unit of length was $(4\pi - 0{\cdot}044799)\, d = 12{\cdot}521569d$ in excess of its attraction at the lower station.

The attraction of the part of the earth inside the surface drawn through the lower station was 69625D at that station, and that of the whole earth at the upper station was

$$\left(69625 - \frac{8}{3}\pi\right)\mathrm{D} + 12{\cdot}5216\, d.$$

Whence $\dfrac{\text{gravity below}}{\text{gravity above}} = 1{\cdot}00012032 - {\cdot}00017984\, d/\mathrm{D}.$

But the pendulum experiments gave

$$\frac{\text{gravity below}}{\text{gravity above}} = 1{\cdot}00005185 \pm {\cdot}00000019.$$

Equating these values we obtain

$$\frac{\mathrm{D}}{d} = 2{\cdot}6266 + 0073.$$

From a detailed examination of the thickness of the strata, and a laborious determination of the specific gravity of each, by Prof. W. H. Miller, d was found to be 2·50. Whence

$$\mathrm{D} = 6{\cdot}566 \pm {\cdot}0182,$$

the last term expressing the probable error of the pendulum results.

Taking into account the ellipticity of the earth and its rotation, Prof. Stokes found a formula which only alters the third decimal place, making,

$$\mathrm{D} = 6{\cdot}565.$$

My impression with regard to this method is that Airy had too much confidence in the accuracy of the pendulum work. The clock was, judging from the description, supported on the same brick floor as the pendulum stand, and small though the transmission of energy from one to the other might be, with so nearly the same rate there might be a mutual effect altering the frequencies. Carlini's arrangement appears decidedly better. The plan adopted of raising the pendulum knife edges from the planes, after each four hours' swing, may have been necessary, but the accuracy of the workmanship would have to be very great to ensure that the contact was the same on lowering again. From Airy's description of a preliminary fault, I should doubt whether the accuracy was sufficient for this.

Examining successive results there is discordance, which appears to bear out the supposition that the rate changed after each lifting up. But when we remember that an error of 0·1 second per day will make an error of more than 1 per cent in the final result, whereas the personal equation with one observer amounted to ½ second in a four hours' swing, it would seem that it is impossible to arrive at the requisite accuracy unless personal equation is quite eliminated. A curious aberration of one or other of the pendulums in one of the series of experiments made afterwards at Greenwich to confirm the temperature correction may be referred to as showing the uncertainty of the pendulum work, though there is no evidence of any such large aberrations in the pit experiments.

Of course, beyond this uncertainty, there is the further and possibly greater uncertainty in the computation of the attraction, through our ignorance of the true density and distribution of the surface strata.

Von Sterneck's Experiments.

(*Nos.* 34, 35 and 39 *in the Bibliography.*)

Major von Sterneck has made some very interesting experiments to determine the variation of gravity underground in the Adalbert Shaft of the silver mine at Pribram in Bohemia and in the Abraham shaft of the silver mine near Freiberg in Saxony. He has also made determinations in a shaft at Krušná Hora in Bohemia in a shaft going nearly horizontally into a hill-side. These experiments are important not only in their results but in the most valuable method used to obtain the ratio of gravity at two stations.

The Pribram Experiments.

The Adalbert shaft is more than 1000 metres deep. In the first experiment made in 1882 three stations were selected, respectively at the surface, about 500 metres below and about 1000 metres below. To determine the change in gravity a single invariable half-second pendulum was used, and this was compared by a modification of the ordinary method of coincidences with a half-second clock carried from station to station with the pendulum. At the surface was a seconds clock rated by astronomical observations, and the coincidence clock was compared with it by a pocket chronometer carried to and fro. The results were not very satisfactory, the values obtained for gravity at the two underground stations being nearly the same, though they differed in level by 456·5 metres. When the values were compared in turn with the surface value, the results obtained for the mean density of the earth were 6·28 and 5·01, the density of the surface strata being taken as 2·75 throughout, a value obtained by determinations from specimens of the rocks.

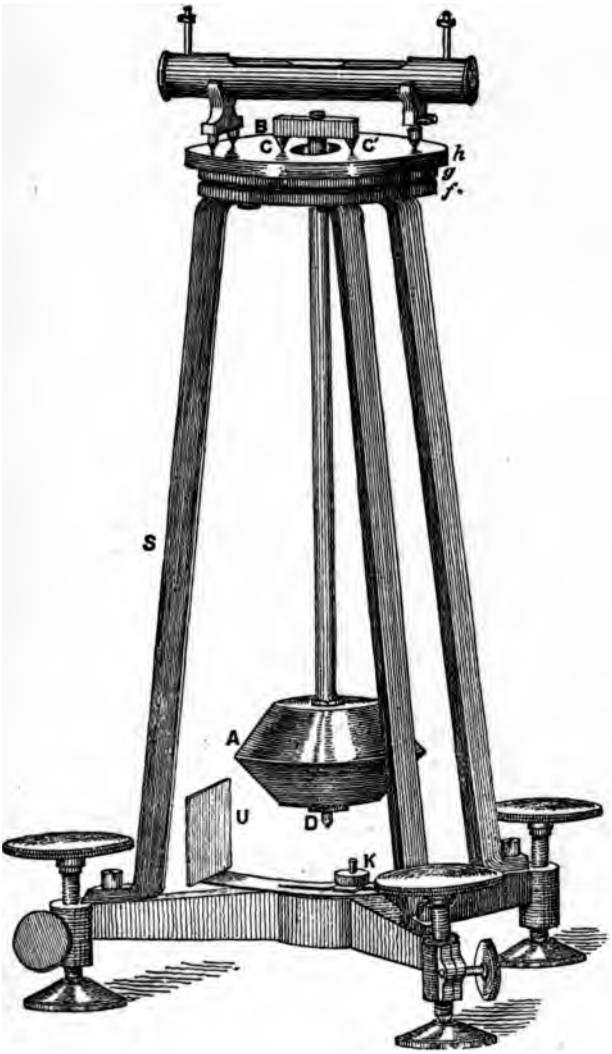
The work of 1882 is to be regarded as preliminary to that of the following year, 1883, when observations were made at four underground stations, including the two of the year before. The surface station was in a cellar under a building about 100 metres from the mouth of the shaft, and the underground stations were all within 200 metres of the vertical through that at the surface. At each station a pillar was built as a stand for the pendulum, about a meter high and 1 × 1½ metres broad at the top.

The pendulum work was on a quite new plan. Two similar invariable pendulums were used, each being a rod (fig. 1) with a heavy weight at its lower end and a knife-edge at its upper end. This was filed away in the middle so that only 2mm. of edge remained at each side. The edges rested on a glass plate with ahole in the middle through which the pendulum could be introduced and set in position. The plate was supported on a very firm tripod stand.

In a determination of gravity one pendulum was always at the surface and the other at an underground station, and the swings *were simultaneously observed in direct comparison with the same clock at the surface by means of an electric circuit.* Thus, for relative values clock errors and variations of rate are of little importance. The clock used beat seconds, and at the beginning of each second it 'closed an electric circuit for half a second. This circuit was carried through all the stations. The tail of the vibrating invariable pendulum moved in front of a scale and was looked at through a telescope. In the focal plane of this was a diaphragm with a horizontal slit in it. The electric current by means of a relay worked a lever carrying a second diaphragm with a slit in it, which moved up and down just in front of the first, the telescope tube being slotted so as to allow the lever to pass in. At the make and at the break, that is, at every half-second the two slits coincided for a moment and the tail of the pendulum was seen. The instant of coincidence was that at which the tail appeared in the middle of its swing when visible through the coincident slits.

When all was ready for observation the pendulums were set swinging, covered with their glass cases and left for ten or fifteen minutes. Then the observers began to note the vibrations. Two sets of five coincidences each, separated by an interval of twelve minutes, were observed in each determination, and four such determinations were made in a day. After this the pendulums were interchanged, and four days were occupied with each underground station. Amplitude, temperature, pressure, and hygrometric state were always observed for purposes of correction.

FIG. 1.



Von Sterneck's Invariable Pendulum.

In finding the mean density of the earth, the formula used was:

$$g = g_s \left(1 + \frac{2h}{r} - \frac{3h}{r}\frac{d}{D}\right)$$

The value of d was taken as 2·75 throughout.

The following table gives the results, the first underground station being omitted since it gave a value of gravity greater than that at the next lower. The temperature is inserted from a later paper:

Station.	Metres above Sea-level.	Metres below Surface.	Temp. R.	Gravity.	Mean Density.
Surface . . .	509	0	—	1000000	
20 Lauf . . .	- 7	516	14·78	461	5·71
26 Lauf . . .	- 239	748	16·71	700	5·81
30 Lauf . . .	- 463	973	19·89	903	5·80
				Mean value	5·77

The method of simultaneous observation in comparison with the same clock, devised by von Sterneck for this experiment, is an immense advance on the old method of comparison with two clocks, and it seems likely to be of great use in the relative determination of gravity at various parts of the earth's surface, now that places far apart are so easily connected by telegraph.

The paper giving an account of these experiments ends with an attempt to determine the variation of gravity within the earth from the Pribram results. Instead of assuming d to be uniform, let us take it as some function of the distance from the earth's centre. It follows that g is also a function of that distance. Assuming that the function has the form:

$$g = ar + br^2 + cr^3,$$

the four stations give four equations to determine a, b, and c. Using the method of least squares to find the most probable values and taking $g=1$ and $r=1$ at the surface, we have

$$g = 2{\cdot}6950\,r - 1{\cdot}8087\,r^2 + 0{\cdot}1182\,r^3.$$

It follows that g increases to a point ·78 radius from the earth's centre, where its value is about 1·06. Then it decreases to 0, when $r=0$. The density at any point can be found from the formula, and is given as

$$d = 15{\cdot}136 - 12{\cdot}512\,r.$$

Of course the author does not mean to put this forward as more than a hint of a possibly available method. Evidently the assumption of a law of density as a function of r alone is untrue near the surface, and even if it were true it would be impossible to obtain with accuracy the values of the constants of the cubic when we only know definitely the values of g at $r=1$, and $r=0$, and have in addition very uncertain knowledge at three points all lying within say $\frac{1}{6000}$ part of the distance between the points $r=1$ and $r=0$. But the method is interesting, and it is conceivable that some day our knowledge may make it fruitful.

The Freiberg Experiment.

In 1871 Prof. Bruhns had found greater values of gravity at the surface than below ground in the Abraham Shaft of the Himmelfahrtsfundgrube silver mine at Freiberg. As this was an entirely anomalous result, von Sterneck determined to re-investigate the values with his two half-second invariable pendulums used simultaneously in the manner which he had devised to eliminate clock differences. As in the later Pribram experiment, the two stations at which gravity was to be compared were connected electrically with a clock sending half-second signals. These worked a relay at each station. But here a change was introduced, for in the relay circuit a spark was made at each signal. This was reflected into a telescope both by a small mirror on the pendulum knife edge perpendicular to the plane of vibration and by a fixed mirror close to the other and parallel to it when the pendulum was at rest. Coincidences were observed therefore, when both reflections appeared on the horizontal cross wire. The temperature and density of air corrections were determined for the two pendulums by actual experiments, varying the conditions first for one and then for the other, and as the results obtained at widely different times were in fair agreement the corrections might be considered known.

In November 1885 Major von Sterneck proceeded to the mine and selected a surface station and four underground stations, the surface and two of those underground being those at which Prof. Bruhns had worked. At each station a brick pillar was built 1 metre high and 60 cm. square, for the pendulum stand. An electric circuit was laid connecting the stations with a clock rated by astronomical observation.

The course of proceeding in taking observations was always the same, and like the course pursued at Pribram in 1883, each of the underground stations being compared in turn with the surface station. When the pendulums were installed, say pendulum No. 1 at the surface and No. 2 below, they were set swinging, covered with their glass cases, and left for an hour before observations were begun. Three groups of eight coincidences with intervals of twelve coincidences between the middle of successive groups were simultaneously observed with each pendulum, amplitude, temperature, and pressure being noted at the beginning and end of a group.

The two pendulums were then dismounted, packed up, and interchanged. They were set up in their new position and left till next day, when another set of observations was made. They were then dismounted, packed up, and then set up again at the same stations without interchange, and after an interval on the same day a third set of observations was made. The pendulums were then interchanged and installed in their original positions, and the next—the third day—a concluding set of observations was made. These four sets, after all corrections were applied, gave a value of gravity above / gravity below. Twelve days sufficed to compare each of the four stations below with the surface, and then a repetition was begun; but unfortunately this was put an end to by the break-down of the electric circuit, and no further work could be done.

The following table embodies the final results:

Station.	Metres above Sea Level.	Metres below Surface.	Mean Temperature R°.	Time of Swing of the Mean Pendulum.	Value of g.	Length of Seconds Pendulum.
Surface . . .	431·93	0	variable	·5009325	1,000,000	994·0000
Stollen. . . .	334·51	97·42	9·83	303	88	88
IV. Gezeugstrecke	174·89	257·04	13·84	245	318	317
VIII. Do.	17·73	414·20	15·37	183	567	565
XI. Do.	-102·15	534·08	18·95	127	793	788

In applying these results to find the mean density it is to be observed that the mine is situated upon a practically even plateau, the small variations in which may be disregarded. The density of the underground strata is very uniformly 2·69.

From the formula:

$$g' = g\left(1 - \frac{2h}{r} + \frac{3h}{r}\frac{d}{D}\right);$$

where g' is surface gravity, g its value at depth, h and d the density of the strata passed through, the following values are obtained by comparing each station underground in turn with the surface:

Stollen . . . . . . D = 5·66
IV. Gez. . . . . . D = 6·66
VIII. ,, ... D = 7·15
XI. ,, ... D = 7·60

This curious result naturally leads to the suspicion that some correction having increasing value with increasing depth is wrongly estimated. The author examines fully the case of the temperature correction, which, if wrong, obviously satisfies this condition, and finds the values of temperature correction for the two pendulums which would give most nearly consistent values of D from the observed results. But the required corrections are decidedly smaller than the mean results of experiments in the preceding years, and further the two sets of observations repeated at stations IV. and VIII. before the break-down, are admirably in accord with the previous observations at the same stations with the old corrections, but are quite thrown out of accord by the new ones. The author therefore comes to the conclusion that a less obvious explanation must be sought, and that there is in fact an abnormal increase of gravity with depth in the locality of the experiment.

Comparing the Freiberg and Pribram results v. Sterneck points out that the increase of gravity in each case is very nearly proportional, not to depth, but to the increase in temperature. The mean value of g below $/g$ at surface is 1·0000624, and the mean underground temperature is 16·60° R. When the separate underground results are compared with the mean, they fall well into line if we assume a temperature co-efficient ·0000091 and a starting-point at 10·25° R, so that

$$g \text{ below } / \, g \text{ above} = 1 + 0{\cdot}0000091\ (t - 10{\cdot}25^\circ).$$

The uppermost station, Stollen, at Freiberg, alone is excepted. Comparing observed values with those calculated from the formula, we have the following table:

—	Station.	Depth below Surface.	Gravity in Terms of Surface Value = 1	Calculated from Formula.	Differences.
Pribram. .	20 Lauf.	516	1·0000461	1·0000459	+2
	26 ,,	748	700	634	+66
	30 ,,	973	903	923	−20
Freiberg .	IV. Gez.	257	318	364	−46
	VIII. ,,	414	567	512	+55
	XI. ,,	534	793	838	−45

Von Sterneck indeed remarks that had he taken the temperature found by Bruhns in 1871 at stations IV. and XI. Gez. he could have calculated the increase of gravity at those stations with an exactness at least equal to that of the actual observations.

This very remarkable result can hardly be ascribed to any direct connection between gravity and temperature. In the first place, we may perhaps fairly compare the Harton pit result with these. The mean underground temperature there was 62.88 °F, say 13·72° R; the value of g below $/g$ above should therefore be 1·00003158, whereas Airy obtained 1·00005185, a difference four times as great as any obtained by von Sterneck in comparing calculated and observed values. Then the various laboratory experiments conducted at various temperatures show no consistent relation. It is true that Baily's values increase with decreasing temperature, but there are other disturbing effects to be allowed for before we can draw any conclusion. On the other hand, Cornu obtained a larger value in summer than in winter, while Jolly could detect no appreciable temperature effect in the decrease of weight with height. I may say also that my own experiments gave no indication of temperature effect. As Major von Sterneck's skill and experience, as well as the consistency of his results, preclude the rejection of the relation he obtains as illusory, we must ascribe it in all probability to underground variation in density, which at the same time affects the difference in gravity above and below and the distance apart of the isothermal surfaces. In our ignorance of the conditions below the lowest level yet reached in mining, there is no difficulty in accepting the explanation.

It is, perhaps, worth while here to note the kind of irregularity in attraction which would be needed to reconcile these underground observations with other experiments.

Let g be the value of gravity at an underground station at depth h, g' its value at the surface above. Then, if G is the gravitation constant, we may put:

$$g = \frac{4}{3}\pi \mathrm{G} r \mathrm{D} + x$$

$$g' = \frac{4}{3}\pi \mathrm{G} r \mathrm{D}\left(1 - 2\frac{h}{r}\right) + 4\pi \mathrm{G} h d + x',$$

where x and x' are the increments due to local variations from the arrangement of the earth in homogeneous concentric spherical shells, and the other letters have their previous significance.

If we divide, to eliminate G, and remember that x and x' are small, we may easily find D in the form

$$\mathrm{D} = \frac{3d}{2 - \frac{r(g - g')}{hg} + \frac{r(x - x')}{hg}}$$

If then $x - x'$ is positive and we neglect it, we shall obtain too large a value of D. And if it increases more than in proportion to the depth—for instance, as h^2, we shall, by neglecting it, obtain an increasingly large value of D as h increases. It does not appear likely that surface irregularities, such as elevations above the upper station, can account for the observed deviations, for in all cases these have been either allowed for or neglected because of their smallness. We can then only turn to underground irregularities and suppose that, in all three mines so far chosen for experiment, there exists below the mine some local increase of density producing a greater attraction at the lower than at the upper station. The peculiar temperature relation found by von Sterneck seems to support this view, for the distance apart of the isothermal surfaces with equally conducting rocks, is almost certainly regulated by conditions prevailing at some depth lower than that we can yet reach, and a variation in their distance apart implies a variation in these conditions.

We must regard as merely accidental the fact that in all three cases the observations point to an abnormal excess in the increase of gravity. Let us apply Stokes's theorem:

$$\int \mathrm{N}\, d\,\mathrm{S} = \mathrm{G} \times 4\,\pi\,\mathrm{M}$$

—where N is the normal attraction over a surface S and M the included matter—to a level area S′ on the surface connected by lines of force from its edge with the edge of a level surface S at an average depth h below. Then if d is the density of the included matter:

$$g'\,\mathrm{S}' - g\,\mathrm{S} = \mathrm{G}\,4\,\pi\,\mathrm{S}\,h\,d$$

or

$$g = g'\,\frac{\mathrm{S}'}{\mathrm{S}} - \mathrm{G}\,4\,\pi\,h\,d.$$

If the lines of force are straight lines proceeding from the earth's centre,

$$\frac{\mathrm{S}'}{\mathrm{S}} = \frac{(r+h)^2}{r^2} = 1 + \frac{2h}{r},$$

and we may take as the normal increase that given by

$$g = g'\left(1 + \frac{2h}{r}\right) - \mathrm{G}\,4\,\pi\,h\,d.$$

But if g increases more rapidly than this everywhere over a large area S, so that on the average

$$g = g'\left(1 + \frac{2h}{r} + \mathrm{K}\right) - \mathrm{G}\,4\,\pi\,h\,d;$$

$$\therefore \quad \frac{\mathrm{S}'}{\mathrm{S}} = 1 + \frac{2h}{r} + \mathrm{K},$$

so that S′ exceeds its normal value by KS, an amount proportional to the square of the linear dimensions. But since a bending outwards of the lines of force from the earth's radius, in passing up from S to S′ adds an area to the normal value of S′ proportional to the product, bending outwards × length of edge, of which the latter factor is proportional to the linear dimensions, it follows that the former factor, or bending outwards, must also be proportional to the linear dimensions. This is contradicted by experience, for there is no cumulative divergence between the astronomical and geodetic normals. Probably, then, if a number of other mines were selected for experiment, we should find some giving abnormal defects and not excesses in the increase of gravity underground.

The true value of experiments on gravity below the earth's surface would appear to lie not in their use to determine the mean density of the earth, but rather in their indication, by anomalies, of irregularities in density in the region round the place of experiment. If, however, it were necessary to depend on this method of determining the earth's density, the application of Stokes's theorem which we have just made points out a way in which the method might be completed and rendered independent of all outside irregularities.

Starting from

$$g' \mathrm{S}' - g \mathrm{S} = \mathrm{G}\, 4 \pi \mathrm{S}\, h\, d,$$

divide by gS, and for g on the right hand put its value $\frac{4}{3} \pi \mathrm{G}\, r\, \mathrm{D}$. Then we get:

$$\frac{g'}{g} \frac{\mathrm{S}'}{\mathrm{S}} - 1 = \frac{3h}{r} \frac{d}{\mathrm{D}}.$$

If, besides observing $\frac{g'}{g}$, we can also measure $\frac{\mathrm{S}'}{\mathrm{S}}$, we may find $\frac{d}{\mathrm{D}}$ without any consideration of matter outside the region between S and S′. But I fear that this only points to an impracticable method of proceeding. It is just conceivable that in some colliery district a suitable ring of pits might be found sufficiently close together. Then at the surface at each pit the normal should be determined, and the variation in normal should be observed in descending the pit, say by noting the variation in level of a mercury trough by observations with a vertical telescope at the top. Thus S and S′ both might be found, and the pendulum work would complete the experiment.

We shall conclude the account of Major von Sterneck's experiments with a brief notice of some pendulum work at Krušná Hora in Bohemia, in 1883. Here there is a nearly horizontal adit to an iron mine on a hill side, and gravity was determined by the Pribram method at the mouth of the adit, at stations along it and at stations at the surface overhead. The investigation hardly comes within our province as directly concerned with the mean density of the earth, but indirectly it is of interest here as affording evidence with regard to the term $3 \frac{h}{r} \frac{d}{\mathrm{D}}$ in the Bouguer-Young formula. Faye had already questioned the necessity for this term, and if we accept Airy's view that mountains are not external additions to a spheroidal earth composed of homogeneous layers but the exposed parts of lighter matter buoyed up and, as it were, floating on the denser substrata, we must agree that the term does not represent the true attraction.

From the values obtained at the surface at different elevations at Krušná Hora, von Sterneck calculates the value of gravity at sea level with and without the term, and he finds that its omission leads to much more consistent values. Later work at the Sagberg, in Hungary, (*Mitth. d. k. u. k. Mil. geog. Inst.*, Band v.), supports this conclusion. The underground values at Krušná Hora came nearly half-way in each case between the value at the mouth of the shaft and the value at the surface immediately above.

All this tends to confirm the conclusion that our knowledge of the distribution of the terrestrial matter is not yet sufficiently exact to enable us to obtain good values of the mean density of the earth from the observed attraction of terrestrial masses. Rather must we assume the mean density from laboratory experiments, and use the observations of terrestrial attractions for the converse problem of determining the distribution of terrestrial mass.

Mendenhall's Experiment on Fujiyama.

(*Nos.* 31 *and* 32 *in the Bibliography.*)

In the year 1880, Professor Mendenhall made determinations of the relative values of gravity at Tokio and at the summit of Fujiyama by means of a Kater's pendulum swung from the upper knife edge only, and therefore used as an invariable pendulum, The time of swing was determined by a method described in the account of an earlier experiment to determine the absolute value of gravity at Tokio (Bibl. No. 31). In this method a "trip-hammer" could be introduced at any instant under the pendulum, so that at the next passage through the lowest point the hammer was "thrown" and an electric circuit was broken. The instant of break was registered on a chronograph on which seconds were marked from a "break-circuit" chronometer. The trip-hammer was introduced at the beginning and at the end of the period of observation only, and did not interfere with the vibrations meanwhile. Thus the beginning of the first and the end of the last swings were accurately recorded, and the number of swings being known the time of a single swing was found. In general, the pendulum swung for thirty minutes, and this length of time was sufficient to give a very accurate determination. With so short a time a very small arc could be used, and as no comparison clock was needed the apparatus was rendered more portable.

At Tokio, the results were all corrected to 23·5° C. by means of an assumed coefficient of expansion, and to a barometric pressure of 30 inches by means of a curve of corrections given by Peirce (*U.S. Coast Survey Report for* 1876, Appendix No. 15). This was determined experimentally by swinging a pendulum in an enclosure at various pressures. The mean result was

$$t_1 = \cdot 999834 \text{ second.}$$

The summit of Fujiyama is much visited by pilgrims, and on it are a number of small huts used by them as temples. A priest allowed one of these to be used for the pendulum work, and the method of determining the time of swing was the same as at Tokio. The chronometer was rated by the aid of a portable transit instrument used by Prof. Chaplin, who accompanied Prof. Mendenhall. The temperature and pressure on the summit varied very little from 8·5° C. and 19·5 inches, and to these values all the observed results were corrected, the mean being

$$t_2 = 1\cdot 000336 \text{ seconds.}$$

From the Tokio result, the sea level value at Fujiyama could be deduced by allowing for the 19′ difference in latitude and the difference in level. It was found to be

$$t_3 = \cdot 999847 \text{ second.}$$

The mountain is very conical, with a nearly constant angle of 138°. It is 2·35 miles high, and from a number of specimens the density was estimated at 2·12, this being arrived at from a knowledge of the density of the specimens when intact and when powdered up so as to exclude the air. With these as data, t_2 and t_3 give the mean density of the earth

$$D = 5\cdot 77$$

Professor Mendenhall, recognising that the density of the mountain is a very uncertain factor, reverses the problem, and assuming Baily's value D = 5·67, he finds the density of the mountain $d = 2\cdot 08$. Had he taken D = 5·5, d would have come out 2·02.

But the method is of course rendered uncertain by the probability that the sea-level value of gravity at Fujiyama, if the mountain were cleared away, would be different from that obtained from the Tokio value. If the Bouguer-Young correction is rejected we can certainly no longer assume that the true sea-level value is the normal spheroidal value. It would doubtless be somewhat less. The attraction of the mountain is probably therefore underestimated, and so the value $\frac{D}{d}$ is over-estimated.

EXPERIMENTS WITH THE TORSION BALANCE.

The Experiment proposed by Michell.

(*See beginning of Cavendish's paper, No.* 8 *in the Bibliography.*)

The experiment carried out by Cavendish and usually named after him, was really devised, Cavendish states, by the Rev. John Michell, who completed an apparatus for the purpose, but did not live to experiment with it.

Michell's plan consisted in suspending in a narrow wooden case a horizontal rod 6 feet long, with a 2-inch sphere of lead hung at each end by a short wire. The suspending wire for the rod was 40 inches long. Outside the case were two lead spheres 8 inches in diameter. These were to be brought up opposite the suspended spheres, one on one side, the other on the other, so that their attractions on those spheres should conspire to turn the rod the same way round. Now moving each large sphere on to the other side of the case so as to pull the suspended sphere with equal force in the opposite direction, the rod should turn through twice the angle which it would describe if the spheres were taken altogether away. Hence half this angle would give the twist due to the attractions in one position alone. Knowing the torsion couple of the suspending wire for a given angle of twist and the length of the rod, the attracting force would be calculable. To find the torsion couple, Michell proposed to set the rod vibrating. From its moment of inertia and time of vibration the couple could be found.

Neglecting all corrections, the mathematics of the method may be reduced to the following.

Let the two suspended balls have mass m each, the two attracting balls mass M each. Let the rod have length $2a$ and with the suspended balls moment of inertia I; let d be the distance apart of the centres of attracting and attracted balls, and let θ be the angle through which the attraction twists the rod.

If μ is the torsion couple per radian twist, and G the gravitation constant, then

$$\mu\,\theta = \frac{2\,\mathrm{G}\,\mathrm{M}\,m\,a}{d^2}.$$

The time of vibration

$$\mathrm{N} = 2\,\pi\sqrt{\mathrm{I}/\mu};$$

whence, eliminating μ,

$$\frac{4\,\pi^2\,\mathrm{I}\,\theta}{\mathrm{N}^2} = \frac{2\,\mathrm{G}\,\mathrm{M}\,m\,a}{d^2}.$$

Now we may obtain another equation containing G by expressing the acceleration of gravity in terms of the dimensions and density of the earth,

$$g = G . \frac{4}{3} \pi \frac{r^3 D}{r^2} = \frac{2}{3} G D C,$$

where r is the radius, C the circumference, and D the density of the earth. Eliminating G between the last two equations and putting for g / π^2 the length of the seconds pendulum L—a useful abbreviation—we find

$$D = \frac{3}{4} \times \frac{L}{C} \times \frac{M m a}{d^2} \times \frac{N^2}{I \theta},$$

where all the terms on the right hand are known or may be measured.

Cavendish's Experiment.

(*No.* 8 *in Bibliography.*)

On Michell's death the apparatus which he had collected for his experiment came into the possession of Prof. Wollaston, who gave it to Cavendish. Cavendish determined to carry out the experiment, with certain modifications; but he found it advisable to make the greater part of the apparatus afresh, though closely following Michell's plan and dimensions.

The actual work was done in the summer of 1797 and the following spring of 1798.

He selected for the experiment, according to Baily, an out-house in his garden at Clapham Common, and within this he appears to have constructed an inner chamber to contain the apparatus, for he states that he "resolved to place the apparatus in a room which should remain constantly shut, and to observe the motion of the arm from without by means of a telescope," in order that inequalities of temperature and consequent air currents within the case should be avoided.

The torsion rod *h h* (Fig. 2, reduced from the figure in Cavendish's paper) was of deal, 6 feet long, strengthened by a silver wire tying the ends to an upright *m g* in the middle. The two attracted balls *x x* were lead, 2 inches in diameter, and hung by short wires from the ends of the rod.

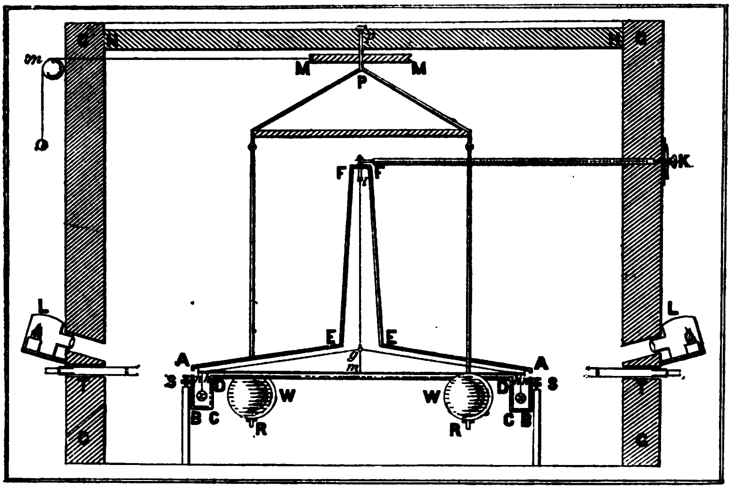
The torsion wire was 39¼ inches long, of silvered copper, and at first of such cross section as to give a time of oscillation about 15m. This was soon changed for one with a time of oscillation about 7m.

The position of the rod was determined by a fixed scale on ivory divided to $\frac{1}{20}$th inch near the end of the arm, the arm itself carrying a vernier of five divisions. This was lighted by a lamp outside the room, and was viewed through a telescope passing through a hole in the wall.

The torsion case was supported on four levelling screws. The attracting masses, lead spheres 12 inches in diameter, W W, hung down from a cross bar, being suspended by vertical copper rods. This bar could be rotated by ropes passing outside the room round a pin fixed to the ceiling in the continuation of the torsion axis.

The masses were stopped when $\frac{1}{8}$ inch from the case by pieces of wood fastened to the wall of the building. When the masses were against the stops their centres were 8·85 inches from the central line of the case.

FIG. 2.



Cavendish's Apparatus.

The method of experimenting varied somewhat, but we may take at random Experiment 13 as an example.

Three consecutive extremities of vibration were observed with the attracting masses in the "negative" position—*i.e.*, the position in which they tended to lower the scale reading. These were meaned in the usual way to give the centre of swing. The masses were then moved round to the positive position, and the next three extremities gave, on meaning, the new centre of swing, and the change in the position of centre was taken as the deflection.

But now a series of observations was made to determine the time of vibration. This was done by noting the instants of passage of two consecutive divisions one on each side, if possible, of the point midway between two successive extremities of swing. By interpolation the instant of passing this midway point was found. Observing several such passages in succession, the intervals were taken as times of vibration. It is true that the point midway between two extremities in a diminishing swing is not the centre of swing, but Cavendish considered that it was reached always sensibly in the same phase of the vibration—that is, at the same time from the instant of rest—and that the interval between two successive passages would give the period, and that this would be freed practically from the effect of progression of the centre of swing. Having thus made several determinations of the period of vibration, the masses were moved back to the negative position, and the difference between the last positive and the first negative centre of swing was taken as another value of the deflection. Several more determinations of time of vibration were then made, and the experiment was brought to an end after lasting about 2½ hours.

The results in this experiment were :

Motion of arm on moving masses from – to + = 6·12
" " " + to – = 5·97
Time of vibration in + position . . 426 secs.
" " – " . . 427 secs.

The accuracy of the time determinations may be judged from the separate results, which were

$$\left.\begin{array}{rlr} 1 & \text{in} & 427 \\ 1 & \text{in} & 424 \\ 3 & \text{in} & 1277 \\ 1 & \text{in} & 424 \end{array}\right\} + \text{position}$$

$$\left.\begin{array}{rlr} 1 & \text{in} & 426 \\ 5 & \text{in} & 2139 \\ 1 & \text{in} & 426 \end{array}\right\} - \text{position}$$

In the + position the centre of swing varied from 23·32 to 23·72, and in the negative position from 17·75 to 17·37.

In computing the results various corrections had to be introduced into the equivalents of the simple formulæ which have been given above. Taking the attraction formula:

$$\mu\,\theta = \mathrm{G}\,2\,\frac{\mathrm{M}\,m\,a}{d^2},$$

a correction had to be made, because the attracting masses were not quite opposite those attracted, as the suspending bar was a little too short. Then allowance was necessary for the attraction on the torsion rod, and a negative correction had to be applied for the attraction on the more distant ball. The copper suspending rods were also allowed for, and a further correction was made for the change in attraction with change of scale reading—*i.e.*, for change of distance between attracting and attracted masses. This correction was proportional to the deviation from the central position, and may be regarded as an alteration of μ.

As to the case it would evidently have no effect when the rod was central, but it was necessary to examine its attraction when the rod was deflected. Cavendish found that in no case did it exceed 1/1170 of the attraction of the masses and therefore neglected it.

Turning now to the vibration formula:

$$N = 2\pi\sqrt{I/\mu};$$

this was correct when the masses were in the "midway" position—*i.e.*, in the line perpendicular to the torsion rod. But when they were in the positive or negative position, the variation in their attraction, as the balls approached or receded from them, made an appreciable alteration in the value of the restoring couple, and thus virtually altered μ. The time had therefore to be reduced by $\delta/185$ of its observed value where δ was the deflection in scale divisions due to the change of the masses from midway to near position.

But it is to be observed that if the weights were moved from one near position to the other, and the time of vibration was taken in either position, then the same correction having to be applied to μ in both formulæ, it might be omitted from both.

In all, Cavendish obtained twenty-nine results with a mean value of

$$D = 5{\cdot}448 \pm {\cdot}033.$$

By a mistake in his addition of the results, pointed out by Baily, he gave as the mean 5·48.

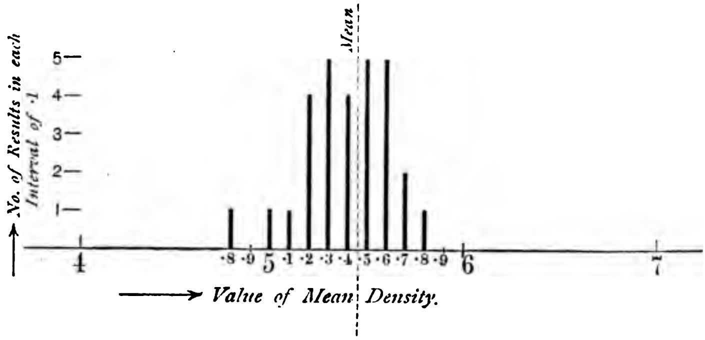
In Fig. 3 I have arranged the twenty-nine results in intervals of ·1 to show the closeness about the mean.

An examination of Cavendish's work in this experiment, fully bears out the general opinion that he was a magnificent experimenter. It is true that details are not given of some of the measurements, an omission adversely criticised by Baily, but I think we may trust to Cavendish's instinct for sound work, and take it for granted that he would not go to so much trouble in the calculations of small corrections if his constants were not known with corresponding accuracy. Of course we can see now that the method might have been improved in some ways, as, for instance, by introducing Baily's method of combining observations in threes, with alternate + − + positions of the masses, so as to eliminate the effect of progressing centre of swing. And the time of vibration might perhaps have been found more exactly, but considering that it was the first attempt to measure exactly forces of such an order the success obtained was most remarkable.

Cavendish himself does not appear to have been satisfied that he had got the best work out of his apparatus, and contemplated a continuation of the experiment, but there is no record that it was ever resumed.

A very important set of observations made in the course of the work, but not bearing directly on the result, remains to be described. Cavendish had noticed that in some cases, where the large masses were left for a considerable time near the small attracted masses, the effect seemed gradually to alter, sometimes in one direction as if indicating an increasing, sometimes in the other as if indicating a diminishing, attraction.

FIG. 3.



Distribution of Cavendish's twenty-nine results about the mean 5·448.

His first idea was that this was due to elastic after-action in the wire, but this was negatived by experiments. He then tested for magnetic effects by replacing the masses by horizontal bar magnets, but none appeared even with such a strong field as the magnets would produce. Finally he concluded that the anomalies were due to air currents through inequalities of temperature produced inside the case by the motion of the masses, which were probably of a different temperature from the case. Heating one side would give rise to an ascending current and a diminished pressure, while the descending current on the other side would be accompanied by an increased pressure. It should be noted that the march of the rod would occur while this current was growing. When the current was steady the march would cease, and there would be a certain steady deflection. When the current began to diminish there would be a return of the rod.

To find the effect of temperature inequalities, Cavendish heated the large masses with lamps, and brought them near the outside of the case containing the attracted masses. In one trial, after half an hour, the outside of the case had risen 1½° Fahr. in temperature, while the attracted masses had been drawn through fourteen scale divisions instead of three as at first, and all the other trials gave a similar effect—a spurious attraction. The masses were cooled with ice in another experiment till 6° or 7° Fahr. below the case. When brought up against the case and left, the rod was found after an hour drawn aside 2½ divisions less than it should have been, or there was apparently a diminishing attraction. To eliminate these effects he took the position of the rod as soon after the removal of the masses as possible, in hopes that the temperature changes would be unable to penetrate the case in time to affect the reading. He concludes these observations with the remark that "in this apparatus, the box in which the balls play is pretty deep, and the balls hang near the bottom of it, which makes the effect of the current of air more sensible than it would otherwise be, and is a defect which I intend to rectify in some future experiments."

It is interesting here to notice that Mr. Crookes found that exactly the opposite effects took place when the air was sufficiently exhausted. He says in his first paper, "On the Action of Heat on Gravitating Masses" (*Phil. Trans.*, 1874, p. 523), "A heavy metallic mass when brought near a delicately suspended light ball, attracts or repels it under the following circumstances:

"1. When the ball is in air of ordinary density.
    a. If the mass is *colder* than the ball, it *repels* the ball.
    b. If the mass is *hotter* than the ball, it *attracts* the ball.
2. When the ball is in a vacuum.
    a. If the mass is *colder* than the ball, it *attracts* the ball.
    b. If the mass is *hotter* than the ball, it *repels* the ball."

A little further on in the same paper he makes a noteworthy suggestion: "Experiment has, however, showed me that whilst the action is in one direction in dense air, and in the opposite direction in a vacuum, there is an intermediate pressure at which differences of temperature appear to exert little or no interfering action. By experimenting at this critical pressure, it would seem that such an action as was obtained by Cavendish, Reich, and Baily, should be rendered evident."

I have said that Baily's method of alternating positions and combining results in threes is superior to Cavendish's, but there is one danger in Baily's method from which Cavendish's is free. With the regularity of operation, all the various parts of the experiment occupy almost exactly the same time each time they are carried out—so at least I found in my own experiment, conducted on Baily's method—and, consequently, a given position of the masses recurs at constant intervals. Thus inequalities of temperature will be propagated through the case in waves of period equal to that of recurrence of any given part of the experiment. Such waves may produce periodic changes in the strength of air current, and if so these will in turn produce a change in the force being measured at any particular phase of the experiment, and different changes for different phases.

In my own experiment I have not observed any positive proof of the existence of periodic disturbances arising from this source. Probably, if they were serious, the first results of a series would always differ from the mean of the series in the same way, but I have not found this to be so. There is a remarkable step-by-step descent of the centre of swing in my observations represented in Diagram 6, which may possibly be due to such a disturbance, but on comparing the results with those two days later, it does not appear to have affected the mean obtained.

Reich's First Experiment.

(*No* 17 *in the Bibliography.*)

In 1837 Professor Reich, of the Academy of Mines at Freiberg, published an account of an experiment which he had made following almost exactly the method of Cavendish.

The apparatus was set up in a large cellar under the Academy, the windows being closed up and the observer being stationed outside the room. The torsion rod was suspended from the ceiling. It was of wood, about 2 metres long, and was enclosed in a box.

The attracted balls were a composition of tin, bismuth, and lead, and weighed 484 grammes each. They were suspended 770 millimetres below the ends of the torsion rod by wires, and hung in tubes depending from the torsion-rod box. The position of the rod was read by a telescope directed through a hole in the door to a mirror at the middle of the rod which reflected a scale near the telescope. A wooden case enclosed the torsion apparatus.

The attracting masses were spheres of lead weighing 45 kilos. each, and they were suspended by long brass wires from little wheels running on rails near the ceiling *parallel to the torsion rod.* They could be moved along by ropes passing from the wheels to the observer outside the door. But as their distances from the balls were different in each of the two positions of each of the two masses, it appeared more convenient to use only one at a time, and to move it from the "null position" opposite the middle of the torsion rod to a position opposite one of the balls, termed "plus" or "minus," according as it increased or diminished the reading.

The moment of inertia of the rod was found by the method applied by Gauss to magnets—*i.e.*, by hanging on equal weights, each about the weight of one of the balls, first from the ends of the torsion rod and then from points half-way along the two arms.

The near distance between attracting and attracted masses varied between 168 and 190 mm. The time of swing was a little over 400 seconds, and the movement of the end of the rod due to the attraction was from 0·6 to 0·8 mm.

After the measurement of distance from mass to ball was made by a kind of horizontal cathetometer, the cellar was closed up for some days before beginning observations in order that the temperature disturbances might subside. When work was to be commenced the torsion rod was set swinging round a suitable part of the scale by moving the torsion head, an adjustment which could be made from outside. Suppose the mass used to be in the "minus" position, then for the first set at least four successive extremities of swing were noted, say a, b, c, d; these were combined in the usual way equivalent to $\frac{1}{4}\ 8\ (a + 3b + 3c + d)$ to give the mean centre of swing. Directly the last reading, d, was taken, the mass was moved to the null position. The transfer was regarded as instantaneous, so that the same reading, d, was taken as the first of the null set. Cornu and Baille have pointed out that this method is open to serious objection, and as used by Baily led to considerable error. An account of the objection will be given in describing Baily's experiment. Taking three more readings, β, γ, δ, the new centre of swing was taken as $\frac{1}{4}(d + 3\beta + 3\gamma + \delta)$. Moving back to the minus position, another centre was found, and so on, the number of centres of swing thus observed in a day varying from 2 to 6. Suppose that four were found, C_1 C_2 C_3 and C_4, Reich took the value $\frac{1}{2}(C_2 + C_4) - \frac{1}{2}(C_1 + C_3)$ as the deflection due to the attraction of the mass.

The time of vibration was determined during the progress of the readings for each centre of swing. It was known by preliminary observations that the centre of swing lay between two given divisions, and the times between successive transits of these two divisions across the cross wire were observed. By interpolation that of the centre of swing could be found. Thus, on June 12 the centre of swing, 74·1750, was previously known to lie between 73·5 and 74·5. The following transits were noted, the second in each case not being used:

Transits of 73·5.	Transits of 74·5.	Intervals between First and Third Transits: 73·5	74·5
XIh. 7m. 7·2sec.	XIh. 7m. 20sec.		
,, 13m. 44sec.	,, 13m. 49sec.	810·8	809·6
,, 20m. 38sec.	,, 20m. 31·6sec.		

By interpolation the interval for 74·175 was found to be 810 secs., whence the time of oscillation was 405·000 secs.

In all, 57 determinations were made on fourteen days in June–August 1837, and the mean result for the mean density of the earth was 5·44. The means of different days varied between 5·1683 and 5·6926.

After the publication of Baily's paper, Reich (*Neue Versuche*, &c., Bibliography No. 21) adopted Baily's method of combination of centres of swing in successive threes to compute the deflection, using $C_2 - \frac{1}{2}(C_1 + C_3)$ and $\frac{1}{2}(C_2 + C_4 - C_3)$. Recomputing the results, he obtained from the same observations the value:

Mean density of the earth = 5·49

Having regard to some criticisms on Baily's experiment by Hearn (Bibl. No. 20), it is important to observe that Reich made a set of observations in which he used a cast-iron attracting sphere of 30 kilos., obtaining a result 5·43, practically agreeing with that obtained when lead was used. This clearly proved the absence of magnetic effect in the experiment with lead, for had there been any such effect with that metal, it would have been enormously greater with iron, and the results could not have been been so nearly equal.

Reich's Second Experiment.

(*No.* 21 *in the Bibliography.*)

Though not in historical order, we may here conveniently describe a continuation of the experiment made by Reich many years later with the same apparatus somewhat modified.

The work was carried out during the years 1847-1850, and the result was published in 1852.

The apparatus had been moved from the cellar to the second floor of the building, where it was under less favourable temperature conditions, but the work was carried out in the new position, the temperature changes being warded off to some extent by covering the case with tinfoil in accordance with Forbes's suggestion to Baily. At the same time electrical action was entirely prevented.

One attracting mass only was used, and this was now placed on a revolving disc or turntable, with its axis immediately under one of the attracted balls. The ball hung in a tube by a long wire from the end of the torsion rod, so that as the turntable revolved the mass could pass freely round the ball. The turntable had the same attraction on the ball in all positions, so that the correction for the suspending wires, &c., of the mass was entirely done away with.

The mode of observing was practically the same as in the first experiment, and the observations were treated by Baily's method.

The results may be classified thus:

No. of Determinations.	Suspension.	Time of Vibration.	Mean Result.
24	Copper wire ·5 mm. diameter, 2270 mm. long	About 700 secs.	5·5712
24	Copper wire 4 mm. diameter, 620 mm. long	About 500 secs.	5·6173
24	Bifilar iron 4·2 mm. apart, 2270 mm. long	About 800 secs.	5·5910

The mean of the means is $D = 5{\cdot}5832 \pm 0{\cdot}0149$.

To test for the presence of magnetic action, ten determinations were made with a diamagnetic attracted sphere of bismuth, with a result of 5·5266, and 12 with an attracted sphere of iron, with a result of 5·6887. Reich suggests that the largeness of this result perhaps indicates a repulsion by the diamagnetic lead mass.

Reich in this, as in his earlier experiment, used the last elongation in one position of the mass as the first in the next position, a method which, as already mentioned, introduces an error, and one which tends to increase the result. But probably Reich would be able to move his masses much more rapidly than Baily, for they were only ⅓ the weight. The error would thus be less, and this may perhaps explain the fact that his result is less than Baily's.

In accordance with a suggestion of Forbes, Reich made some interesting attempts to determine the attraction from the time of swing alone.

If N is the time of swing when the attracting mass is away altogether, N′ the time when it is in position at one side at a distance E (from centre to centre) and is pulling the ball a distance A towards itself, then it is easily shown that

$$N' = N\left(1 + \frac{A}{E}\right).$$

If N″ is the time of swing when the mass is placed in the vertical plane through the torsion rod and is pulling the ball outwards, it can be shown that

$$N'' = N\left(1 - \frac{A}{2E}\right),$$

$$\therefore\quad N' : N'' = 1 + \frac{A}{E} : 1 - \frac{A}{2E}.$$

If two masses be used at opposite ends of a diameter their effects are doubled, and we have:

$$N' : N = 1 + \frac{2A}{E} : 1 - \frac{A}{E},$$

whence
$$A = \frac{N' - N''}{N' + 2N''} E.$$

But A is equal to $a\theta$ in the formula of p. 42, and we therefore do not require a deflection experiment.

The values of $\frac{N'}{N''}$ were very discordant, but the mean value gave

$$D = 6{\cdot}25.$$

Baily's Experiment.

(*No.* 19 *in the Bibliography.*)

The most extensive series of determinations yet published are those made by Baily at the instigation of a Committee of the Royal Astronomical Society in the years 1841–2 after several years of preparation and trial.

The apparatus, which was generally like that of Cavendish, was set up in a room in Baily's house in Tavistock Place. The window, which faced north, was doubly glazed and covered with paper, and the chimney was stopped up to prevent draughts. The torsion box was fixed to the ceiling, and was in shape like an inverted hollow T. The attracting masses were at the two ends of a plank rotating on the top of a pillar, rising from the floor under the torsion box. The torsion box, the plank and the masses, were all enclosed in a large wood case resting on the floor.

The position of the torsion rod was read by a telescope, mirror, and scale, the mirror being fixed at the centre of the rod. The telescope and scale were fixed in one corner of the room, and ropes passed to this corner from the central pillar. By these the observer at the telescope could turn the masses round.

The masses were of lead, about 12 in. in diameter, most carefully cast and turned into true spheres. They weighed about 380 lbs. each, and were 80 inches apart on the plank. Their positions relative to the torsion box were fixed by stops against which the plank moved, so that there was about 11 inches between the centres of the attracting and attracted masses.

The torsion box was lined inside with foil, then wrapped in flannel, and then surrounded by an outer wood case gilded outside. The gilding was due to a most valuable suggestion of Professor Forbes, who advised it in order that radiation from outside bodies might be in great part reflected. Temperature changes would thus penetrate the case much more gradually. At the same time all risk of electrical disturbances was abolished, for the torsion rod moved within a closed conductor.

The torsion rod was of deal, 77 inches long and braced like Cavendish's, but the balls were fixed at the ends with the axis of the rod passing through their centres. The rod generally used varied somewhat in weight at different seasons, even as much as from 2300 to 2400 grains, but this would have exceedingly little effect on the result.

The peculiarity of Baily's experiment consisted in the number of different balls and different suspensions which he employed.

Thus he used 2½-inch lead, 2-inch lead, 2-inch zinc, 2-inch glass, 2-inch ivory, 1½-inch platinum, 2½-inch hollow brass and finally a heavy torsion rod with no balls at all. For suspension he used bifilar silk different distances apart, bifilar brass, bifilar iron, and also single copper wires of different diameters. The suspending wires were usually 60 inches long. The time of vibration varied from 100 seconds to 500 seconds.

An experiment was generally conducted in the following method:

On beginning a day's work it was usually necessary to increase the oscillation of the rod by moving the attracting masses to one side and the other alternately. Suppose that when the swing was large enough, the masses were in the positive position—*i.e.*, the position in which their attraction increased the reading—then the positive end of the swing was noted, 108·90 in the example below, and the masses were immediately moved round to the negative position.

As in Reich's experiment, it was assumed that this transfer was practically instantaneous, and that 108·90 might serve as the first reading for the vibrations in the negative position. Three more were then read as recorded in the example below, and immediately on the completion of the last reading, 80·80, the masses were turned back to the positive position, and 80·80 was taken as the first of a new four for that position. This process was carried on usually for some hours with alternate + and − positions, with three readings in each, made up to four in the way described.

The four readings were then combined in the usual way, which is equivalent to $\frac{1}{4}(a + 3b + 3c + d)$, to give the mean centre of swing. Thus in Experiment 32 the results were:

Position.	Extreme Points.	Centres of Swing.	Mean.
	108·90		
Negative	82·60	95·975	95·862
	109·80	95·750	
	80·80		

The mean centre of swing in the next experiment, No. 33, with the positive position, was 113·350, and in the following, No. 34, negative, it was 95·875. Then the deviation at the time of Experiment 33 was taken as:

$$\tfrac{1}{2}\left(113\cdot350 - \frac{95\cdot862 + 95\cdot875}{2}\right) = 8\cdot741.$$

Similarly, the deviation in Experiment 34 was taken as:

$$\tfrac{1}{2}\left(\frac{(33) + (35)}{2}\right) - 34,$$

and so on.

This method was devised to eliminate the effect of progressive motion of the centre of swing which was almost always taking place, and on the assumption of the uniformity of march the method is successful.

The time of vibration was determined during the progress of each single experiment by Reich's method. Taking Experiment 32 again, it was known by preliminary observation that the centre of swing lay between 95 and 96. The times of transit of these two divisions across the cross wire were noted for three successive transits, though the middle transit was not used and is omitted below.

EXPERIMENT 32.—OBSERVED TIMES OF TRANSIT.

	Division 95.		Division 96.
	10 26 34		10 26 23
	10 41 41		10 41 31
2 vibrations in	15 7	in	15 8
1 vibration in	7 33·5	in	7 34

Whence, by interpolation, the time of transit of the centre of swing 95·862 was determined as 453·931. This was used as the time of vibration.

As the centre of swing was usually moving in one direction—in 32 and the following experiments in the positive direction—this method would not give quite a right result. If, for instance, the passages noted were in the same direction as the progressive motion of the centre of swing, successive passages would occur at earlier and earlier phases of the vibration, and so the calculated period would be too small. But if the passages were in the opposite direction, it would be too large. Now in successive experiments the transits were in opposite directions, and so by combining the results in threes—*i.e.*, by taking $\frac{1}{4}(t_1 + 2t_2 + t_3)$ as the time of vibration in the experiment from which t_2 was deduced—this effect was largely eliminated.

During every experiment the distances between the masses in their near position and an assumed central point were measured by means of plumb-lines and microscopes, and the mean of these was employed. The mathematical investigation justifies the use of this mean, and indeed a little consideration renders a special mathematical investigation needless. For suppose that in the +

position the distance from ball to mass is less than the mean by x, the attraction is greater than at the mean distance d by $\frac{2x}{d}$ of its value. On the negative side it is less than at the mean distance by the same fraction, and the mean value of the attraction corresponds to that at the mean distance.

In all, Baily made over 2000 determinations with various suspensions, attracted balls, and torsion rods; in each position of the masses observing the centre of swing and measuring the time of vibration and the distances.

The mathematical discussion of the vibration and deflection was contributed by Airy, who took into account variations in the restoring force due to variations in attraction of mass and of enclosing case and other fixtures. But since these equally affected the square of the time of vibration and the mean deflection due to the attraction of the mass in the two positions, they did not affect the result. It therefore only remained to compute the moment of inertia of the torsion system and the corrections to the attraction couple $\mathrm{M}\,m\,a/d^2$ due to the planks, to the rod, etc. and using the values found, the formula on page 42 only needs modification by using a more correct value for the value of gravity in terms of the earth's dimensions, and by taking into account ellipticity and rotation.

It should be observed that the weights were corrected to vacuum. But as Routh pointed out, the weights in the air should have been used, since the masses attract the air as well as the balls. The difference, however, would be very small. If the weights used in the original weighings were correct in air, the maximum correction would be required, and even then it would not be more than 1 in about 9000 for lead, though greater for glass and ivory.

The final result was:

$$\text{Mean density of the earth} = 5{\cdot}6747 \pm {\cdot}0038.$$

I have entered somewhat into detail in describing Baily's methods of observation because there are some curious anomalies in the results which he obtained, probably to be largely explained by his methods.

The most remarkable anomaly, which evidently was a source of much trouble to Baily himself, though he never found any explanation, was the decrease in value of the mean density with increase in weight of attracted balls. Cornu and Baille (C. R. 86, p. 699) point out that this is probably due to error arising from the assumption that the last elongation for one position of the masses may be taken as the first for the succeeding position. Let us suppose that the mass is in the positive position. After taking the last reading, which is always at the positive end, the mass is moved round. But the rod has really started on its return journey, and it is only after an interval that the attraction of the mass begins to help the torsion of the wire to pull the rod back. The result is that the rod arrives at its centre of swing with less momentum than it would have, did the attraction act throughout—a momentum corresponding to a lower starting-point— and the deduced centre of swing is too high. On the other hand, when the mass is moved from − round to + immediately after a reading at the negative end, the momentum acquired during the return corresponds to a shorter swing—*i.e.*, to a higher reading—and the deduced centre of swing is too low.

Thus, from the uniformity of Baily's method, the deduced centre of swing is always too high in the negative position and too low in the positive. Rejecting therefore the first reading in every series of 4 recorded by Baily, error from this source should be eliminated. Taking ten examples at random of experiments which appeared to be regular among themselves and which included lead, zinc, brass, ivory, and the rod alone as attracted masses, Cornu and Baille found that on rejecting the faulty reading the mean density was reduced from 5·731 to 5·615.

Reducing Baily's general mean in this ratio, it becomes 5·55. But to make the most of Baily's work it would be necessary to go through the whole of his experiments and correct them all from this point of view.

It is not quite evident how this error should vary, as it seems to do, with the weight of the balls. The time of vibration with the lighter balls would be less for a given suspension, and the time occupied in turning the masses round would be a larger fraction of the whole time, but I suspect that more than this is required to explain the variation. For we should expect that with given balls the result would depend much more evidently than it does on the time of vibration. It appears possible that the sudden withdrawal of the mass led to a reduction of pressure on that side of the case, and a consequent impulse to the ball opposing its motion.

Another very curious anomaly, a relation between the temperature and the value of the mean density, has been pointed out by Hicks (*Proc. Camb. Phil. Soc.*, vol. v. p. 156). It appears that if the results are arranged in order of the temperature where they were obtained, the mean density falls almost uniformly as the temperature rises. I have not examined this point thoroughly, but by taking out the experiments made with balls of one kind alone, as with the 2-inch lead ball, it would appear that the relation is then not quite so regular. I suspect it is partly due to the fact that a great number of the ivory and glass experiments were made in the cold weather of the winter and spring, and that the summer experiments were chiefly with heavy balls.

It is perhaps worth while pointing out that Baily did not take into account the effect of air resistance in altering the period of his torsion rod. I have examined several experiments taken at random and, using the observed decrement of swing, I have found that the time is reduced in some cases by 1 in 1000, which would give a mean density result wrong by 1 in 500. But this error is trifling in comparison with that already noticed.

As to Baily's method of conducting the observations there is one more obvious remark. He took a long step backwards in observing from the same room and in approaching the instrument every few minutes to make his microscope measurements. It was unfortunate, with Cavendish's example before him, that he did not interpose a more efficient screen between the instrument and so large a source of disturbance as himself.

For many years this experiment was generally regarded as saying almost the last word on the subject. But the critical examination which it has received in later years has entirely destroyed any confidence in the result. It remains, however, as a most remarkable and useful example of the danger of substituting multiplication of observations for consistency.

The Experiment of Cornu and Baille (still in progress).

(*Nos.* 25–28, *and* 36 *in the Bibliography.*)

In 1870 MM. Cornu and Baille announced their intention of repeating the torsion experiment to determine the mean density of the earth. In 1873, they published a note in the *Comptes Rendus* (No. 20 in the Bibliography) describing briefly the apparatus constructed, and the results obtained up to that point. They commenced their work by a careful study of the torsion balance and of the best mode of guarding it from disturbances. They were led to the conclusion that the moment of the air resistance is accurately proportional to the velocity. They then proceeded to the construction of the gravitation apparatus, and in order to vary the conditions they made it as different as possible from that used by their predecessors Cavendish, Reich, and Baily.

The apparatus was set up in a cellar in the École Polytechnique at Paris. The torsion rod was a narrow aluminium tube 50 cm. long. The balls were of copper 109 grms. each, and the position of the rod was read by the reflection from a central mirror of a scale 5·6 metres distant. The suspending wire was of annealed silver, 4·15 metres long and such that the time of a complete vibration was 6 min. 38 sec. The attracting mass consisted of the mercury filling a hollow cast iron sphere 12cm. in diameter, the mercury being pumped from a sphere on one side to another on the other side, to avoid the vibration and air disturbance, due to the motion of heavy masses as used in previous methods. The weight of each sphere of mercury was 12 kilogrammes.

To avoid counting time, a method of electric registration was adopted, so that a graphic record was obtained giving the complete law of movement.

In July and August 1872, they obtained a mean result, 5·56, and in the following autumn and winter another mean result, 5·50. The small difference between the means they ascribed to a flexure of the rod which diminished its moment of inertia. The first set was regarded as the more trustworthy.

In the paper describing their experiment, they point out the remarkable decrease of Baily's results with increasing weights which they ascribe to an error in his calculations of the attraction of the rod. They estimate that his mean value would be reduced to 5·55 if the weights were so heavy that the rod was negligible in comparison. As we have seen, they found a key to the anomaly later.

In *Comptes Rendus*, vol. lxxxvi., 1878, they published three short notes on the continuation of their work and on the torsion balance. In the first they describe satisfactory tests of the accuracy of the law that air resistance is proportional to velocity, and they find that for a velocity of 1 cm. / sec. the resistance is about ·012 dynes / sq. cm.

In the second note they state that they have improved the gravitation apparatus by adding two more spheres of mercury and by reducing the distance of the spheres from the balls. The balance had been so perfected that for more than a year the time of vibration remained at 408 seconds within a few tenths. The mean of the results obtained was again 5·56.

It is in this paper that MM. Cornu and Baille point out the probable explanation of Baily's result of increasing density with diminishing weights, which I have already described.

The third note relates to the solution of the small oscillation equation when the restoring force has a term proportional to the square of the displacement. I pass over this, as it has no bearing on work yet published.

I have not found any later publication relating to the gravitation experiment directly, though I believe that it is now almost concluded and that the result is very near 5·5.

Boys's Experiment (still in progress).

(*No.* 42 *in the Bibliography.*)

A most valuable contribution to the theory of the torsion experiment was made by Professor Boys in a paper in the *Proceedings of the Royal Society*, vol. xlvi., 1889, p. 245.

In this paper for the first time, a calculation is made as to the most suitable dimensions for the apparatus. It is evident that, if the deflection can be maintained large, every diminution in the size of the apparatus down to the limits of accurate measurement is a clear gain, for the smaller the apparatus the more easily is it kept at one temperature, and the smaller are the convection currents produced when given inequalities exist, It is from this point of view that Boys takes up the subject. He says that "the sensibility of the apparatus is, if the period of oscillation is always the same, independent of the linear dimensions of the apparatus. Thus, if there are two instruments in which all the dimensions of one are n times the corresponding dimensions of the other, then the moment of inertia of the beam and its appendages will be as $n^5 : 1$ and, therefore, the torsion also must be as $n^5 : 1$. The attracting masses, both fixed and movable, will be as $n^3 : 1$ and their distance apart as $n : 1$. Therefore the attraction will be as n^6/n^2 or $n^4 : 1$, and this acting on an arm n times as long in the large instrument as in the small, therefore the moment will be as $n^5 : 1$, that is, in the same proportion as the torsion, and so the angle of deflection is unchanged.

"If, however, the length of beam only is changed, and the attracting masses are moved until they are opposite to and a fixed distance from the ends of the beam, then the moment of inertia will be altered in the ratio $n^2 : 1$, while the corresponding moment will only change in the ratio $n : 1$, and thus there is an advantage in reducing the length of the beam, until one of two things happens; either it is difficult to find sufficiently fine torsion thread that will safely carry the beam, and produce the required period, and this, I believe, has, up to the present time, prevented the use of a beam less than half a metre in length, or else, when the length becomes nearly equal to the diameter of the attracting balls, they then act with such an increasing effect on the opposite suspended balls, so as to tend to deflect the beam in the opposite direction, that the balance of effect begins to fall short of that which would be due to the reduced dimensions if the opposite ball did not interfere."

He then points out that a better effect may be obtained with a short beam (or torsion rod) if the centre of the attracting is not opposite to that of the attracted mass, but such that the line joining the centres makes an acute angle with the prolongation of the beam. Under certain assumptions the best position is calculated.

But the difficulty of contrary moment on the more distant mass may be avoided, he shows, by having the two halves of the torsion rod at different levels and the attracting masses also at these different levels.

Further, his discovery of the method of making fine quartz fibres puts it within our power to have "a sufficiently fine torsion thread that will safely carry the beam and produce the required period."

The author then describes the beautiful instrument by which the torsion gravitation experiment, at least in the qualitative form, is rendered available even as a lecture experiment.

The torsion box is replaced by a vertical brass tube fixed on a brass base. The attracted masses are cylinders of lead 11·3 mm. long and 3 mm. in diameter, fixed by light brass arms to a light glass tube so that their axes are 6·5 mm. distant from the axis of the glass tube, and their centres are at levels differing by 50·8 mm.

A small mirror on the upper end of this tube is opposite a window in the brass tube, and the whole torsion arrangement is suspended by a quartz fibre such that the period is 80 seconds.

The attracting masses are two cylinders of lead 50·8 mm. in diameter and of the same length, fastened to the inside of a wide brass tube surrounding the narrow one and turning round its axis. A lid in two halves covers this outer tube, and the whole keeps at a very equal temperature.

In experiments with this instrument, Boys found that his anticipations were most satisfactorily verified, and he stated his intention of making another apparatus of larger dimensions, so that all the quantities could be measured to 1 in 10,000.

This apparatus has since been constructed, and the experiment has, I believe, been carried out at the Clarendon Laboratory at Oxford. The result is not yet published.

EXPERIMENTS WITH THE COMMON BALANCE.

The balance method of experiment consists in determining the increase of weight of a mass hanging from one arm of a balance, when a large attracting mass is placed below the suspended mass, so that its attraction is added to that of the rest of the earth.

The two attractions, that of the mass and that of the earth, are therefore directly compared. Knowing the dimensions and distance of the earth, and knowing the dimensions, distance, and mass of the extra attracting body, the mass of the earth may be determined. Thus, if the mass of the earth is E, and that of the attracting body is M; if the distance of the earth's centre from the attracted body is r, and if that of the attracting body's centre (supposing it a sphere) is l; if the weight of the attracted mass is m grammes and the observed increase is w grammes, then,

$$\frac{E}{r^2} : \frac{M}{l^2} = m : w,$$

whence
$$E = \frac{r^2}{l^2} \times \frac{m}{w} \times M.$$

If we regard the problem as a determination of the mass of the earth, this appears to be the most direct method.

Unfortunately it suffers from practical disadvantages which are referred to in the accounts of the various experiments.

Von Jolly's Experiment.

(*Nos.* 29 *and* 33 *in the Bibliography.*)

In 1878, Prof. v. Jolly published an account of experiments which he had made in his laboratory at Munich, to determine the variation of weight with height above the earth's surface. A balance was fixed 5·5 m. above the laboratory floor, and its case was surrounded by another case lined inside and out with silver paper. The position of the beam was determined by the reflection of a scale from a central mirror, viewed by a telescope at a distance. Under each scale-pan the case was pierced, and to the pan was attached a wire passing through the hole and supporting a second scale pan 5·29 metres lower. The wires were boxed in, and the lower pans hung in cases below. Two nickelled brass kilogrammes were first balanced against each other in the upper pans, and then one was removed to the lower pan, nearer to the earth's centre by 5·29 metres. It consequently gained in weight.

The gain observed after corrections for the air displaced, which differed in the two positions, was 1·5099 mgm. as the mean of 10 experiments, the greatest difference from the mean being ·08 mgm.

From the formula $g' = g\ (1 - 2h/r)$ the gain should have been 1·6822 mgm. The author ascribes the difference to local configuration, as the Physical Institute at Munich is at a lower level than other parts of the town. He states, in conclusion, that he intends to repeat the experiment, now building up a lead sphere of separate bars under the lower pans, and determining the increase of weight with and without the sphere. He thus proposes to determine the mean density of the earth.

In 1881, v. Jolly published an account of a completion of this experiment, which he carried out in a tower of the University at Munich. A staircase went round the walls on the inside of the tower, leaving a square hole 1·5 metres wide down the middle, and at the top the balance was fixed.

This was constructed to carry loads of 5 kgs. on each side. Its knife edges were steel, the plates agate, and its position was read as before by mirror and scale. From each pan a wire of gilded brass hung in a zinc tube and supported another pan at its lower end, 21 metres below.

It was found that difference of temperature in these tubes produced great disturbances, especially if they were connected below. They were, therefore, wrapped in straw, and the lower pans were in separate cases.

The weights consisted of two globes containing 5 kgs. of mercury each. Four globes were, however, prepared of equal weights and equal volumes, and when two were filled with mercury all four were sealed up. To begin with, the two heavy globes were placed above, and the two light ones below, and balanced without any attracting mass below the lower ones. Then the two on one side only were interchanged and the gain in weight observed. By the use of the four globes any variation in air density the same at the same level was without effect.

The best work was on days with small temperature changes, still air, and overcast sky. When the disturbances were great work was not attempted. The experiments were always begun at 9 A.M., as the temperature was the most uniform in the morning. Ten determinations of the centre of swing were made with the weights above, the beam being lifted and released after each determination, and the mean was taken as the equilibrium position. The pair of globes on one side were then interchanged, and at 11 A.M. ten more determinations were made as before, an additional weight of 29·967 mgm. being necessary to bring the balance near to its former position, and again the mean was taken.

The sensibility of the balance being known, the difference between the positions of equilibrium in scale divisions could be converted to milligrammes, and this was added to 29·967 to give the increase in weight.

In all, fifty determinations were made between July and October 1879, and the mean result was a gain of 31·686 mgm., the separate results ranging through about ·8 mgm.

Calculation from the formula $g' = g\ (1 - 2h\ /\ r)$, gives a difference of 33·05 mgm. The smallness of the observed result may be due to high ground near, attracting at the lower position upwards more than at the upper. This would reduce g more than g' and bring them nearer together.

A lead sphere was now built up of bars, below the lower scale pans. Its radius was 0·4975 metres, and its weight 5775·2 kg.

The weighings above and below were then carried on as before, fifty determinations being made between November 1879 and July 1880.

The mean gain in weight was 32·275 mgms. with a difference of about ·4 mgm between extremes.

The increase due to the attraction of the lead was therefore 0·589 mgm. The distance of the centre of the mercury globe from that of the lead sphere was 0·5686 metres. Using the formula on p. 61, modified to

$$\frac{4}{3}\pi r^3 D = \frac{r^2}{l^2}\frac{m}{w}\frac{4}{3}\pi b^3 d, .$$

where l is the distance ·5686 m., b the radius, and d the density of the lead sphere, Von Jolly obtained as the earth's mean density:

$$D = 5·692 \pm ·068.$$

As Richarz has pointed out (*V. J. S. der Ast. Gesellschaft Jahrgang* 24, Heft 1, p. 22), Jolly has here made a trifling arithmetical mistake, only affecting the last figure, and he has neglected the spheroidal and rotation corrections affecting the same figure.

But with so much uncertainty in the third figure it is not worth while revising the calculation in order to correct only the next figure.

Von Jolly thought that possibly the large variation might be due to hygroscopic properties of the glass. He therefore substituted metal globes empty, and filled with lead, but with no improvement.

When the difficulties of weighing accurately in an ordinary balance case are considered, we must recognise that Jolly, working with a case 20 metres high, obtained very consistent results for the gain in weight. But the value of the mean density of the earth did not depend on the total gain in weight, but on the difference in gain with and without the sphere, a quantity of only half a milligramme. The ranges of error of the two gains were respectively ·8 mgm. and ·4 mgm, so that some determinations with the sphere present were very near some when it was absent, and there was even overlapping in two cases. The quantity to be measured was in fact too small for the method employed. The temperature differences between the spheres and their surroundings, after the interchange, could hardly subside in two hours, and doubtless large disturbances were produced by the interchange.

One observation was made by v. Jolly which is worthy of especial note. His experiments were conducted at different times of the year with large range of temperature, but without any apparent effect on the gain in weight when the mass was removed from the upper to the lower station.

König and Richarz's Experiment (still in progress by Drs. Richarz and Krigar-Menzel).

(*No.* 37 *in the Bibliography.*)

In 1884 Drs. König and Richarz independently conceived a method of using the balance, which, while adopting Jolly's principle, should increase the quantity to be measured, and decrease the disturbances.

The method consisted in first measuring the variation of gravity with height after the manner of Jolly, but with a greatly reduced height, and then in inserting a large block built up of lead between the upper and lower levels with vertical borings for the passages of the wires. Suppose now that two weights balance at the upper level. One, say the right, is moved below. The lead block pulls it up instead of down, and a small weight, w, must be added to restore equilibrium. If, on the other hand, the left weight is moved down, w must be added to the left pan. If we begin then with the right weight down and the left weight up, and then change to right up and left down, we must take w from the right and add w to the left, a total change of $2w$. But w when corrected for variation of weight before the lead block is in position, is twice the attraction of the block. The difference observed, then, is four times the attraction.

Drs. Konig and Richarz, considering from preliminary calculations that the method was very promising, arranged to carry it out together, using a nearly cubical lead block about 2 metres high.

An earth-covered casemate in the citadel of Spandau was placed at their disposal by the German Minister of War, and the balance was installed there. Dr. König, on his appointment to a professorship in the University of Berlin, retired from the work, and his place was taken by Dr. Krigar-Menzel, who is now working at the experiment in conjunction with Dr. Richarz. Until a few months ago the difficulties in determining the best mode of working, the best form of knife-edge, and the variation in weight with change of height had not been fully overcome; but from a letter which I received last March (1892) from Dr. Krigar-Menzel, I learn that the separate weighings of a kilogramme agree now within ·02 mgm. of the mean, and that the determinations of $g' - g$

for a change in height of 2·3 metres lie between $64/10^8$ and $67/10^8$ of the whole, whereas

$$\frac{g-g'}{g}=\frac{2h}{r}=\frac{71}{10^8}\text{ nearly.}$$

a difference to be ascribed to the thick walls round the locality.

The Author's Experiment.

This is a balance method. The first published account was given in 1878, in the paper No. 30 in the Bibliography; but as the experiment in its final form is fully described in Part II. (pp. 72 *et seq.*) no further reference need be made to it here.

EXPERIMENTS WITH THE PENDULUM BALANCE.

Wilsing's Experiment.

(*Nos.* 40 *and* 41 *in the Bibliography.*)

We may regard this method of experiment as most nearly allied to the torsion method, the torsion rod being replaced by a pendulum rod with balls at each end, and supported in a vertical position by a knife-edge just above the centre of gravity. The torsion couple is then replaced by the gravity couple formed by the weight and the pressure against the knife-edge. Using attracting masses, one on one side of the upper and the other on the other side of the lower ball, the pendulum is deflected. On now lowering one mass and raising the other through the length of the pendulum the deflection is reversed, and the deflection is double that due to the masses in one position alone. The value of the couple may be determined by taking the time of swing, and the torsion formulæ on p. 41 may be considered as applicable with the new interpretation of $\mu\theta$, the restoring couple.

In the apparatus devised by Wilsing, the pendulum was a brass tube 1 metre long, 4·15 cm. in diameter, and 0·16 cm. thick. In the middle were two slits through which passed the support for the pendulum knife-edge. The knife-edge was of steel, 6 cm. long and firmly attached to a frame-work fixed to the brass tube. At the ends of the pendulum were the two attracted balls, brass (Rothguss) spheres, each about 540 gms. Small movable perforated brass discs were placed on a pin projecting vertically upwards from the top of the upper ball. These were for a purpose which will be indicated below.

The experiment was conducted in a cellar of the Potsdam Astrophysical Observatory. On a foundation three feet deep a brick pillar was built, and to one side of this a cast-iron bracket was attached carrying the support for the agate plate on which the knife-edge rested. A protecting case covered with tin surrounded the pendulum, and was provided with a window through which could be seen a mirror fixed to the pendulum just above the knife-edge. The observer was outside the cellar and viewed with a telescope through a hole in the door the reflection of a scale let into the door. The scale was on opal glass lighted from behind.

The attracting masses were cast-iron cylinders 325 kg. each, with their axes horizontal and directed to the pendulum rod. They were connected by a wire rope passing over a pulley near the ceiling, and they moved up and down in guides, coming against stops on the guides when they were exactly level with the balls. The pulley could be rotated from the outside by a suitable contrivance, and so the weights could be moved by the observer. Their axes were obliged to be inclined each at about 15° to the plane of vibration, since the bracket supporting the pendulum did not allow of their near approach when their axes were in that plane.

Before commencing work Wilsing made an interesting observation on the effect of a vessel filled with snow, placed outside the pendulum case in the plane of vibration, and found that it produced a steadily increasing deflection, which might be ascribed to the warping of the pendulum through unequal temperatures on the two sides.

The regular observations were conducted in the following manner: The masses being in one position the pendulum was lowered on to the agate support and its swinging was commenced. Four elongations gave the centre of swing. The masses were then moved to the other position, and four elongations gave the new centre. Then they were moved back, and four more elongations were observed. Then again to the second position, and four more elongations were observed. This completed a set, giving a value for the deflection. Meanwhile the times of swing were observed in each case by noting the times of transit of two scale divisions near to and one on each side, if possible, of the centre of swing. If the swings were not decreasing, the difference between two times of transit of a given division in the same direction would give the time of a double vibration. But, through the decrement of amplitude, the time observed would not be quite correct. By observing the times of transit in the opposite direction also, a time of double vibration would be obtained, affected with an opposite and nearly equal error. The mean of the four times obtained with the two sets of transits of the two divisions was taken as the true time.

The time of swing was then reduced by the removal of one of the small discs from the upper ball, and its value determined by observing a number of transits at the beginning and a number at the end of an interval of some minutes. The logarithmic decrement was so small as to be negligible in its effect, so that it was possible to observe transits in one direction of one division only. From the times it was easy to deduce the period. A second disc was then removed, and the new and still shorter period determined in the same way. The discs were then replaced and the experiment was complete. Usually another was made later in the same day, and in all there were thirty-seven sets of observations.

In Wilsing's first paper is a very full mathematical treatment of the subject, discussing first the effect of the cylindrical form to which the so-called knife-edge really approximates, and the expression for the time of swing is found when this and buoyancy of air and attraction of masses are taken into account.

Then he shows that by removal of one of the discs and a new determination of the time of swing the value of the restoring couple may be found without a knowledge of the moment of inertia.

Neglecting all corrections, we may at once see the principle of this. The ordinary pendulum formula gives the time T_1 by

$$T_1^2 \times M g f = \pi^2 I,$$

where $M g f$ is the restoring couple per radian (supposed proportional to the angle) and I is the moment of inertia. Removing a mass m distant a from the knife-edge on the upper side, $M g f$ is increased by $m g a$, while I is diminished by ma^2. The time T_2 is given by

$$T_2^2 \times (M g f + m g a) = \pi^2 (I - m a^2).$$

Subtracting,

$$(T_1^2 - T_2^2) M g f = \pi^2 m a^2 - m g a T_1^2,$$

whence,

$$M g f = \frac{\pi^2 m a^2 - m g a T_2^2}{T_1^2 - T_2^2}.$$

The removal of a second disc gives another value of the same quantity, and in the actual observations the two values were in very satisfactory agreement.

After a careful and laborious investigation of the moment of the attraction, the author discusses the effect of amplitude on time of swing, theory indicating that the ordinary reduction formula must be replaced by

$$T = T_0 + x T_0^3,$$

and x is shown by experiment to have a value $a \phi - b \phi^2$, where ϕ is the amplitude, and a and b are certain constants.

Returning to the simple torsion rod formula of p. 42:

$$D = \frac{3}{4} \cdot \frac{L}{C} \cdot \frac{M m a}{d^2} \cdot \frac{N^2}{I\theta},$$

we must replace $\frac{N^2}{I}$ by $\frac{\pi^2}{Mgf}$, where Mgf has the value found above.

But this formula must now be corrected. In the first place, the value of Mgf has to be found in a form taking account of form of knife-edge, attraction of masses, and buoyancy of air, and T_1 and T_2 must be corrected by the new method to infinitely small arcs. Then, $\frac{M\,m\,a}{d^2}$ has to be replaced by the value of the attraction couple, taking into account the fact that the mass is a cylinder, and that it attracts rod as well as balls.

The value obtained from the experiment was

$$D = 5{\cdot}651 \pm 0{\cdot}017.$$

There was, however, reason to doubt the value assigned to the attraction moment on the rod without the balls. These were therefore removed, and the experiment repeated on the rod alone, the result of thirty-one sets being

$$D = 5{\cdot}731 \pm 0{\cdot}020.$$

Calling these values D_1 and D_2, we may put

$$D = D_1 + \frac{x}{m_1} \; ; \; D = D_2 + \frac{x}{m_2},$$

where x is the error in the value of the moment of attraction and $\frac{1}{m_1}$ and $\frac{1}{m_2}$ are the co-efficients of the moment in the two values for D. They are therefore known.

$$\text{Hence} \qquad D = \frac{m_1 D_1 - m_2 D_2}{m_1 - m_2}$$

$$= 5{\cdot}594 \pm 0{\cdot}032.$$

In the second paper, the author describes a repetition of the experiment made after certain improvements in the apparatus. The pendulum support was re-arranged so that the axes of the masses were in the plane of vibration. Greater precautions were taken against temperature changes; among others, the window was blocked up, the door was lined with tin plate, and the masses were covered with tinfoil. All the constants were also carefully redetermined.

Three new series of experiments were now made, twenty-six sets with the original balls, thirty-nine with lead balls, each weighing about 745 grammes, and forty-two with the pendulum *rod alone.*

It was then assumed that there was an error in the value of the attraction moment, the same for the same deflection in all three cases. For the first and third series, eliminating the error as before, the result was

$$D = 5{\cdot}556 \pm 0{\cdot}026.$$

For the second and third,

$$D = 5{\cdot}584 \pm 0{\cdot}015,$$

and from both these,

$$D = 5{\cdot}588 \pm 0{\cdot}013.$$

Giving this and the previous result $5{\cdot}594 \pm 0{\cdot}032$ weights proportional to their probable errors, the final result is:

$$D = 5{\cdot}579 \pm 0{\cdot}012.$$

It might appear possible that the use of iron attracting masses was injurious through magnetic action. The author, in the first paper, shows clearly that this magnetic effect must be negligible. For in certain sets of observations one of the brass discs was replaced by an iron one of the same weight without any trace of alteration in the observations.

Laska's Proposed Experiment.

(*No.* 43 *in the Bibliography.*)

The only notice of this method which I have seen consists of a few lines in the journal referred to above. The method appears to differ from Wilsing's chiefly in two points: First, the attracting mass is the mercury filling a glass globe, and this can be withdrawn or poured in at pleasure. In the figure accompanying the description there is only one globe near the ball at the bottom of the pendulum rod, but of course it would be easy to arrange one on each side of each ball. Secondly, the position of the pendulum is read by an interference band method. The interference layer is the air between a plate on the upper end of the rod and a fixed plate close to it. The motion of the rod alters the thickness of the layer, and shifts the bands just as in Fizeau's instrument for measuring expansion.

The author describes this apparatus as having been set up, but no details are given in the note as to the quality of its performance. Indeed, I gather that it is only a trial installation, and has not been arranged to give absolute results.

SUMMARY OF RESULTS HITHERTO OBTAINED.

Approximate Date.	Experimenter.	Method.	Result.
1737–40	Bouguer	Plumb-line and Pendulum	Inconclusive
1774–6	Maskelyne and Hutton	Plumb-line	4·5 to 5
1855	James and Clarke	„	5·316
1821	Carlini	Mountain Pendulum	4·39 to 4·95
1880	Mendenhall	„	5·77
1854	Airy	Mine Pendulum	6·565
1883	Von Sterneck	„	5·77
1885	Von Sterneck	„	about 7
1797–8	Cavendish	Torsion Balance	5·448
1837	Reich	„	5·49
1840–1	Baily	„	5·674
1852	Reich	„	5·583
1870–	Cornu and Baille	„	5·56–5·50
1889	Boys	„	in progress
1879–80	Von Jolly	Common Balance	5·692
1878–90	Poynting	„	5·493
1884–	König, Richarz and Krigar Menzel	„	in progress
1886–8	Wilsing	Pendulum Balance	5·579
1889	Laska	„	in progress

PART II.

A DETERMINATION OF THE MEAN DENSITY OF THE EARTH AND THE GRAVITATION CONSTANT BY MEANS OF THE COMMON BALANCE.

(From the *Philosophical Transactions*, vol. clxxxii. A, 1891, pp. 565–656.)

I. ACCOUNT OF APPARATUS AND METHOD.

In a paper printed in the *Proceedings of the Royal Society*, No. 190, 1878 (vol. xxviii. pp. 2–35), I gave an account of some experiments undertaken in order to test the possibility of using the common balance in place of the torsion balance in the Cavendish experiment. The success obtained seemed to justify the intention expressed in that paper to continue the work, using a large bullion balance, instead of the chemical balance with which the preliminary experiments were made.

The apparatus for the experiments now to be described was first set up in the Cavendish Laboratory at Cambridge through the kindness of Professor Clerk Maxwell. After spending some months in working at the experiment, but without much success beyond the detection of some sources of error, I left Cambridge, and ultimately the apparatus was again set up at the Mason College, Birmingham. The difficulties in carrying out the work with any approach to exactness have been far greater than were anticipated, and many times work has been begun and results have been obtained, but examination has shown them to be affected by large errors which could be traced and eliminated by further improvements in the apparatus.

At the beginning of 1890, however, the apparatus was brought into fair working order, and during the course of the year I made a number of experiments with the results recorded in this paper.

The Principle of the Experiment.

The object of the experiment, in common with all of its class, may be regarded, primarily, as the determination of the attraction of one known mass M on another known mass M′ a known distance

d away from it. The law of universal gravitation states that when the masses are spheres with centres d apart this attraction is GMM'/d^2, G being a constant—the gravitation constant—the same for all masses. Astronomical observations fully justify the law as far as M'/d^2 is concerned. They do not, however, give the value of G, but only that of the product GM for various members of the solar system.

To determine G we must measure GMM'/d^2 in some case in which both M and M′ are known, whether they be a mountain and a plumb-bob, as in Maskelyne's experiment, the surface strata and a pendulum-bob, as in Airy's experiment, or two spheres of known mass and dimensions, as in all the various forms of Cavendish's experiment.

Knowing the gravitation constant G, we may at once find the mean density of the earth Δ. For if V be the volume of the earth —regarded as a sphere of radius R—the weight of any mass M′, being the attraction of the earth on it, is

$$G\,V\,\Delta\,M'/R^2.$$

But if g is the acceleration of gravity the weight is also expressible as $M'g$.

Equating these we get

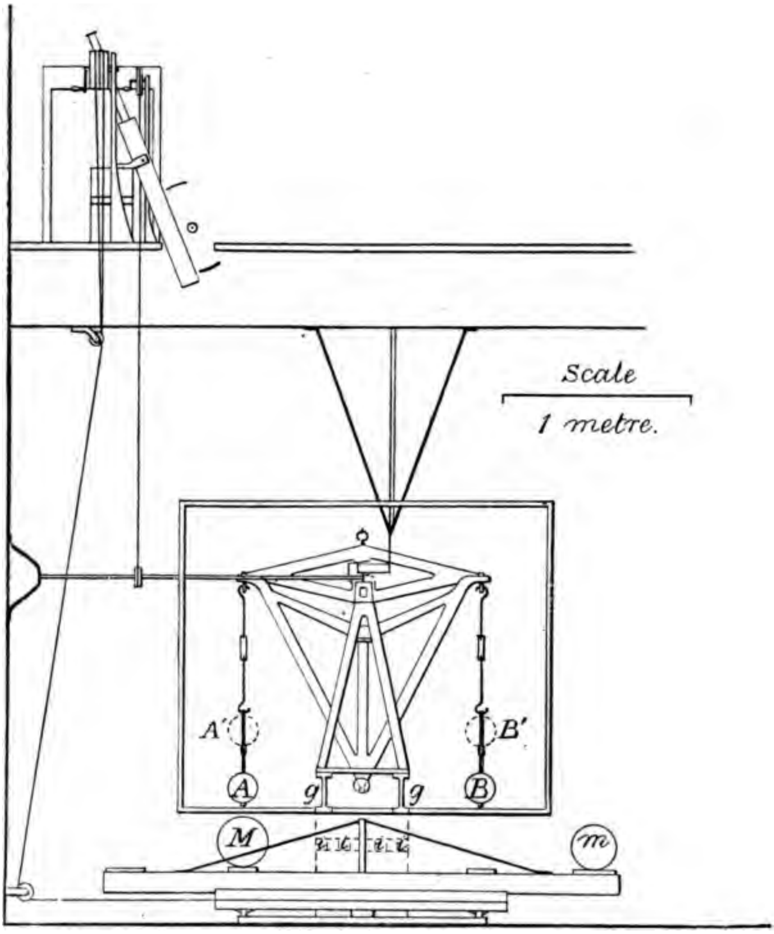
$$\Delta = g\,R^2/G\,V.$$

Method of using the Common Balance.

In using the common balance to find the attraction between two masses, perhaps the most direct mode of proceeding would consist in suspending a mass from one arm of a balance by a long wire, and counterpoising it in the other pan. Then bringing under it a known mass, its weight would be slightly increased by the attraction of this mass. The increase would be the quantity sought if the attracting mass had no appreciable effect before its introduction beneath the hanging mass, and if, when beneath it, the effect on the balance could be neglected. This is very nearly the principle of the method used by von Jolly, and it is that of the method used in the preliminary experiments referred to above, in which a mass of 453 grms. of lead was hung from one arm of a chemical balance (about 40 centims. beam) by a wire 1·8 metres long, and was attracted by a mass of 154 kilogrms. of lead. But the attraction to be measured was exceedingly small, rather less than 0·01 milligrm., and it therefore appeared advisable to use a much larger balance with a larger hanging mass so that the attraction might be made comparable with the weight of exactly determined riders. Other anticipations as to proportionate increase of sensibility and diminution of effect of air currents, have hardly been justified in the way I expected, though, by the ultimate form of the apparatus, they have, I think, been more than realised.

With increase in the length of beam, a differential method

FIG. 4.



Elevation of balance room and observing room. The front of the case is removed, and the front pillar is not shown. The pointer and mirrors are at the back.

became applicable, by means of which the attraction of the mass on the beam was eliminated, and the necessity for prolonging the case to allow of a long suspending wire was removed. This will be seen from a consideration of Fig. 4. Let AB represent equal masses suspended from the two arms of the balance, and let M be the attracting mass put first under A, the position of the beam being noted. If M is then placed under B its attraction is not only taken away from A but added to B, so that the tilting of the beam is that due to nearly double the attraction to be measured. Of course there are what we may term *cross-attractions*, in the first position, of M on B, and in the second position, of M on A, but these may be allowed for in the calculations. We cannot give any mathematical expression for the attraction of M on the beam and suspending wires, owing to their irregularity of shape. But this attraction is eliminated if a second experiment is made in which A and B are raised equal known distances to A′ and B′. For the *difference* between the two increments of weight on the right is due solely to the alteration of the positions of A and B relative to M, the attraction on the beam remaining the same in each. From the observed effect of a known alteration of distance the attraction at any distance can be found.

This is, shortly, the method adopted. The arrangement was ultimately complicated by the addition of a second mass *m*. Originally the mass M was alone on a turntable which revolved about a vertical axis immediately under the central knife-edge of the balance. And some experiments which I made led me to suppose that mere change of position of the mass did not affect the level of the balance. However, after a complete determination in 1888 of the mean density, when I supposed that the work was finished, an examination of the results showed some curious anomalies, which I could only ascribe to a tilting of the whole floor on the displacement of the mass. Making new tests as to the effect of removal of the mass, I found that the previous tests had been quite wrong in principle, and that there was a very appreciable effect quite visible in the telescope when the masses A and B were removed, and M was removed from one side to the other, the slope of the floor changing by an angle comparable with a third of a second. If this had been absolutely constant in amount, the differential method would have eliminated it; but, probably, it varied slightly in successive motions of the turntable, and the results showed that there was also a secular change, the amount of tilt gradually increasing. This secular change was probably due to increasing rigidity of the floor, so that it tilted over bodily, moving the supports of the balance with it, an increase partly due, perhaps, to the pressure of the building, which had only been erected ten or twelve years, but chiefly, I think, to a gas engine recently erected next door. When this was doing heavy work, the vibrations were very plainly felt, and no doubt they greatly aided the floor in "settling down." A second balancing mass *m* was therefore added, half as great as M, and on the opposite side of the turntable, but twice as far from the axis. The resultant pressure was now always through the axis, and I could detect no tilting of the floor when the turntable was moved. Of course the balancing mass acted somewhat to reduce the effect of the larger attracting mass, but in a calculable ratio.

Finally, in order to eliminate or reduce the effect of any want of symmetry in the moving parts or in the masses, a second set of experiments was made with all the masses turned over and moved from left to right, and the mean of the first and second set was taken.

I now proceed to a detailed description of the various parts of the apparatus and the mode of experiment.

The Balance Room.—The balance room is in the basement of the Mason College, immediately under my room, and about 20 metres from the street. On one side were three windows looking on to a small courtyard, entirely surrounded by high buildings, but the windows have been bricked up. On the two adjacent sides are two other rooms, and on the opposite side a closely fitting door opening on a short corridor with doors at each end. There is no chimney in the room, and only an opening in the ceiling through which the balance was observed from the room above. The floor is of brick, resting on earth, and is very firmly laid.

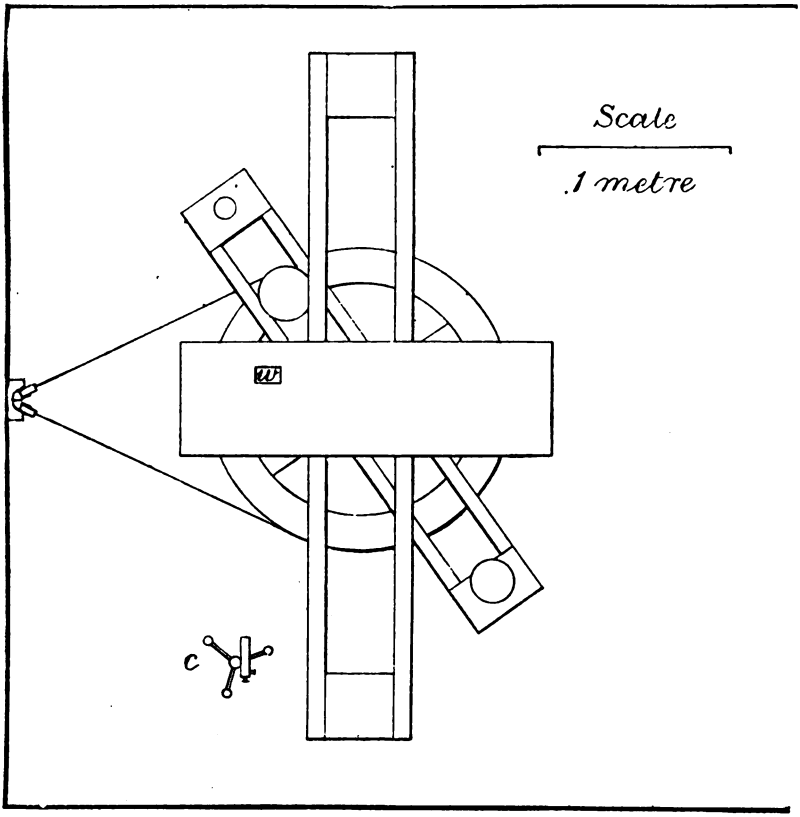
The temperature of the room was taken by means of a thermometer with a protected bulb at the end of a long wooden rod hanging down from the room above. The thermometer was about 6 feet from the floor, near one end of the case, and it could be rapidly pulled up into the room above and read by the observer before its temperature sensibly varied. The temperature never appeared to vary so much as 0·1° C. in the course of two or three hours.

The Balance Case and its Supports.—The case (Fig. 4) is a large cabinet of 1¼ inch wood, 1·94 metre high, 1·63 metre wide, ·61 metre deep, with three large doors in front giving access to the hanging masses and riders, and a small door at the back near the mirror hereafter described. It is lined inside and out with tinfoil, and under each of the suspended masses is a double bottom with a layer of wool between, making a total thickness of about 1½ inch or 4 centims. At the top is a small window about 10 centims. square, through which the oscillations of the beam were observed. On each side within the case are placed three horizontal partitions, like shelves, to hinder circulation of the air.

The larger attracting mass and the attracted masses are gilded, and it is possible that some advantage may arise from having the surface of the case of different metal. For if it, too, were gilded, it would readily absorb radiation from the large mass, and when the inside temperature changed, the suspended masses would readily absorb radiation from the inner surface of the case. But gold probably absorbs considerably less of tin radiation than it absorbs of gold radiation, and so temperature changes are probably lengthened out more than if the case were gilded.

It was necessary to support the case so that the attracting masses could be moved about underneath it, and also to make it independent of the floor. Two brick pillars, 58 centims. × 36 centims. and 56 centims. high, were therefore built on thick beds of concrete under the floor, and about $3\frac{1}{2}$ metres apart. They rise up free from the brick floor. Stretching between them are two parallel iron girders (*g*, *g*), about 30 centims. apart, and with their under side 56 centims. above the floor. The balance case is placed across the middle of these girders (see plan, Fig. 5), with its under surface level with that of the girders. The square base plate of the balance is placed on the girders on three levelling screws. Two horizontal screws attached to the girders bear against each edge of the base plate, so that it can be adjusted and fixed in any position.

FIG. 5.



Plan of turntable, girders, pillars, and balance case. *w*. Window in case. *c*. Usual position of cathetometer.

To lessen vibration one tier of bricks is removed from each pillar, and in its place are inserted eight cylindrical blocks of india-rubber (*i*, *i*, Fig. 4), originally 7·5 centims. diameter and 7·5 centims. high. These crushed down almost 1 centim. at once, but have not shown any further measurable contraction in the course of several years. Their effect in deadening vibration has been surprisingly great.

The Turntable.—On a bed of concrete, and quite free from the brickwork of the floor, is a circular rail of cast iron, 1·3 metre in diameter. On this, on conical brass wheels and pivoted at the centre, runs the turntable, about 1·5 metre in diameter. This is made of wood and covered with tinfoil. It is like a wheel with a flat circular rim, and with four flat spokes arranged as a cross. It is as nearly symmetrical as possible, and at opposite ends of a diameter are placed two shallow cups, in either of which the large attracting mass may rest. The centres of these cups are a distance apart equal to the length of the balance beam. There are slots cut through the bottom of each cup, so that the bottom of the mass can be seen for the purpose of measuring the vertical diameter.

Two beams, 2·74 metres long, run across the turntable 26 centims. apart, with the cups between them, and across the ends are two boards, each with a circular hole 12 centims. in diameter, and in either of these the smaller, or balancing mass, may rest. These beams are braced by brass rods to brass uprights at their middle points to diminish bending.

The turntable is moved by an endless gut rope passing round it, and fixed at one point of the rim. The two sides of the rope pass over pulleys on to a drum in the room above. There are stops on the circular rail, against which come brass pieces on the turntable when the masses are in position at either end of the motion. The drum can be turned easily by the observer at the telescope. Since the knife-edges and planes of the balance are of steel, all the moving parts of the apparatus are free from iron. As an illustration of the necessity of this, I may mention that for some time I used what I supposed to be a brass wire rope to move the turntable, but on looking out for the explanation of some irregularities I found that the brass was wrapped round a core of steel wire, which acquired poles at the highest and lowest points in the position in which it always rested between different sets of weighings. These poles had quite an appreciable action on the balance beam.

The Balance.—This is of the large bullion balance type, with gun-metal beam and steel knife-edges and plates. It was made specially for the experiment by Mr. Oertling, with extra rigidity of beam. Its performance has shown the great excellence of the design. The central knife-edge is supported on a steel plate by a frame-work rising 107 centims. above the base plate, and the usual movable frame can be raised or lowered from outside the case, fixing the beam or setting it free to oscillate. The beam has often been left free to oscillate for months at a time, with the full load of 20 kilogrms. on each side, but I have no reason to suppose that the knife-edges have suffered at all.

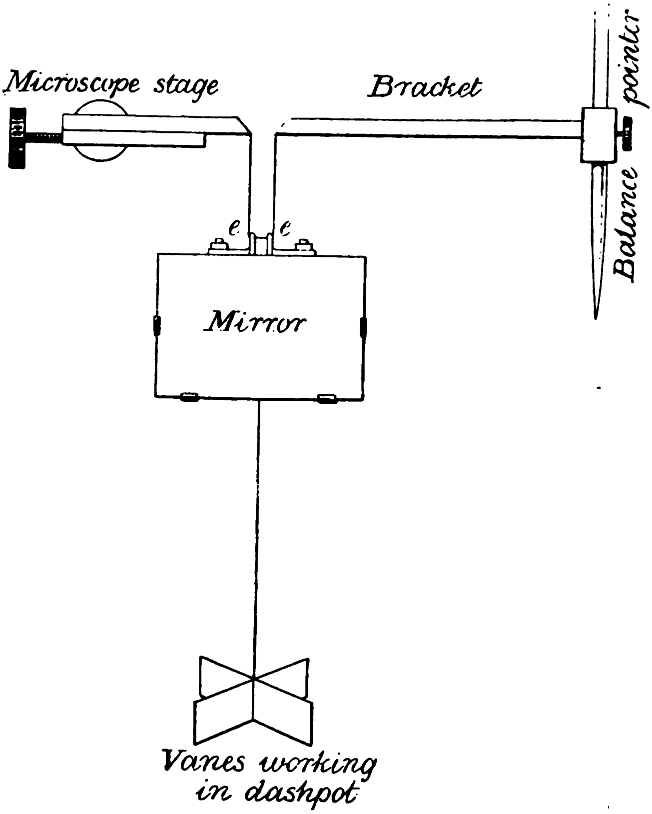
The length of the beam was measured by taking the length of each half separately by a beam compass, and the mean of several measurements gave 123·329 centims. as the total length. The standard scale used throughout was that of a cathetometer made by the Cambridge Scientific Instrument Company. This scale has been verified at the Standards Office, and taking its coefficient of expansion as $\frac{1}{50000}$, it may be regarded for our purpose as perfectly correct at 18°, any errors being at that temperature much less than the errors of experiment. Comparing the beam compass with this scale, it was found that ·06 centim. must be subtracted, reducing the length to 123·269 centims. Now both beam and scale are of gun-metal and may, therefore, without serious error, be assumed to have the same coefficient of expansion, so that this is the length of the beam at 18°. At 0° it is 123·232 centims.

Mirrors, Telescope, and Scale.—At first a mirror was attached to the centre of the beam and the reflection of a scale in it was observed, either in the ordinary method or in the method (*Roy. Soc. Proc.*, No. 190, 1878), where a second fixed mirror is used to throw the ray of light a second or even a third time back on to the moving mirror, each return increasing the deflection of the ray. But it was then necessary to make the time of vibration very long, and even when the time was three minutes, the tilt due to the attraction—*i.e.*, the change of resting-point—did not amount to more than two or three scale divisions. Now certain irregularities observed when the apparatus was first set up at Cambridge, led to experiments on the time taken by heat to get through the case in sufficient quantity to affect the balance, and I found that a coil of copper wire placed close under the case on one side (the bottom of the case being then solid, 1 inch thickness), heated by a current yielding 100 calories per minute, began to produce an appreciable disturbance on the balance in about ten minutes, doubtless by the creation of air-currents from the heated floor of the case. It appeared advisable, therefore, to reduce the time of a complete experiment to less than this if possible, and, consequently, the time of a single swing very much below three minutes. This could only be done if at the same time the optical sensibility were very greatly increased.

The employment of what may be termed the double-suspension mirror method due, I believe, to Sir William Thomson, and used by Messrs. G. H. and Horace Darwin in their experiments on the Lunar Disturbance of Gravity (*Brit. Assoc. Rep.*, 1881), has very satisfactorily solved the problem, giving a greatly increased deflection on the scale, even when the time of oscillation is as short as twenty seconds.

This method, which deserves to be more generally known and applied for the detection of small motions, consists in suspending a mirror by two threads, one from a fixed point, the other from the point which moves. The angle through which the mirror turns for a given motion of the latter point is inversely as the distance between it and the fixed point, so that by diminishing this distance the sensibility of the arrangement may be almost indefinitely increased.

FIG. 6.



Double suspension mirror (half size).

To apply it to the balance, a small bracket (Fig. 6) is fixed to the ordinary pointer of the balance, about 60 centims. below the central knife-edge. This projects horizontally at right angles to the axis of the beam, and it is bevelled at the edge. Close to it is another bevelled edge attached to a microscopic stage movement which is fixed on to the central pillar of the balance. A thread of silk (as supplied for the Kew magnetometer) is fastened to the stage, passes over the bevelled edge, through two eyes *e e*, on a light frame holding the mirror, up over the bevelled edge of the bracket, and is fastened to the bracket. The microscope stage movement allows the distance between the threads to be adjusted and also enables the azimuth of the mirror to be altered.

Of course, if the mirror were weightless, it would not affect the sensibility of the balance, and the threads might be brought very close together. But the weight of the mirror—it is silver on glass, 56 millims. × 38 millims. × 10 millims.—has a considerable effect on the sensibility, diminishing it with decrease of distance between the points of suspension. In practice it has been found convenient to work with the threads parallel, and from 3 to 4 millims. apart, the time of swing one way being adjusted to about 20 seconds. A less time hardly suffices for a correct determination and record of the scale reading. Taking four millims. as the distance, and supposing the bracket to be 600 millims. below the knife-edge of the balance, the mirror evidently turns through an angle 150 times as great as that through which the beam turns.

The drawback to this method of magnification is that the mirror has its own time of swing and is easily disturbed. The swings of the mirror and the disturbances are, however, effectually damped by having four light copper vanes attached to the end of a thin wire projecting down from the mirror and working in a dashpot with four radial partitions not quite meeting in the centre, one vane being in each compartment. I found that mineral lubricating oil is very suitable for the dashpot, as the surface keeps quite clean and there is little evaporation. The swings of the balance are also very greatly damped by this arrangement, but the effect of this will be discussed later.

The telescope and scale are in the room over the balance-room (see Fig. 4), a hole being cut through the floor, and a small glass window fixed in the top of the case. As the suspended mirror is in a vertical plane it is necessary to have an inclined mirror fixed in front of it to direct the light from the scale horizontally on to it and back again to the telescope. With the magnification used it was necessary for good definition to have an exceedingly good inclined mirror, and several were rejected before a suitable one was obtained. That finally used is a silver on glass oval mirror, 60 millims. × 40 millims., by Browning. The glass window in the case is optically worked and carefully adjusted to be normal to the path of the light.

The telescope has a 3-inch object glass of about 4-feet focal length. It is fixed on a brick pillar built on one of the brick arches, which form the ceiling of the balance room, and it rises free from the floor of the observing room. To destroy vibration one course of bricks is replaced by blocks of india-rubber. The scale has 50 divisions to the inch (say $\frac{1}{2}$ millim.), ruled diagonally, and divided to tenths by cross lines. It is photographed on glass from a scale drawn on paper with very great care, 50 inches long (say 127 centims.), and with 500 divisions. The photograph is $\frac{1}{10}$th of this length, and only the central part of the scale, about 60 divisions in length, has been used. The diagonal ruling enables a tenth of a division to be read with certainty, and the readings recorded in the Table, pp. 129 *et seq.*, are in tenths. Though the lines appear somewhat coarse, I have not been able to find another scale equal to it in distinctness and in ease of reading.

As all the results depend on the ratio of measurements, taken almost simultaneously, of deflection due to attraction and rider respectively, in the same part of the scale, I have not thought it necessary to calibrate it.

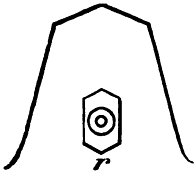
The scale is fixed horizontally on the end of the telescope close to the object glass with a piece of ground glass over it. It was illuminated in general by an incandescent lamp placed above it, once by an Argand burner.

The distance from the scale to the mirror and back is about 5 metres. It follows that 1 division of the scale corresponds to an angular motion of the mirror through ·0001 radian. But this is at least 150 times the angle through which the beam turns for the same deflection. So that 1 scale division implies an angular motion of $\cdot 000000\dot{6}$ radian, or $\frac{2}{15}''$ in the beam. As the total length of swing in Table III. is never more than 12 divisions, the angular vibrations of the beam are at the most about $1''\cdot 6$, and the linear vibrations of the masses, since the half beam is about 60 centims., are at the most about ·005 millim. This shows that it is quite unnecessary to consider any change of distance due to vibration. The greatest deviation from the mean in any of the series of weighings recorded is about one per cent. of the rider value, corresponding to about $\frac{1}{10}$ of a division, or an angle of $\frac{1}{75}''$ in the beam, and a distance of ·00004 millim., say $\frac{1}{600000}$ inch, in the motion of the masses. This seems to show that the method is accurate as well as sensitive.

Determination of the Value of the Scale Divisions by Means of Riders.—This was done by means of centigramme riders (Fig. 7), these being the least weights which appeared capable of sufficiently accurate determination. Instead of transferring the same rider from point to point, it was much easier to use two equal riders, and to take one up while the other was being let down a given distance from it. The distance selected was about 2·5 centims., since the deflection due to the transfer of one centigramme so far along the beam was nearly equal to that due to the greatest attraction to be measured.

At first the riders when on the beam rested in **V** notches in a pair of parallel brass strips fixed on and parallel to the beam.

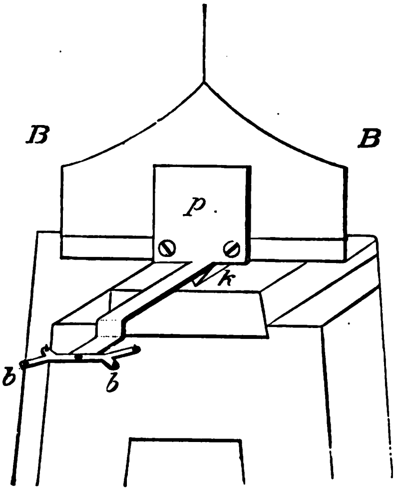
FIG. 7.



Rider, actual size, and end of lifting rod, *r*.

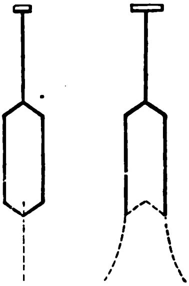
But this plan was soon abandoned, as there was no certainty about the position of the rider in the notches. The riders were then supported in little wire frames, each hung by two cocoon fibres from the edges of a plate fixed to the beam, the edges being parallel to the central knife-edge. The only objection to this method was the very considerable time spent in replacing the fibres after the breakages which occurred on dusting, or any readjustment of the balance.

FIG. 8.



Subsidiary rider beam, *bb*, attached to centre of balance beam, *BB*, by plate *p* just above central knife-edge, *k* (half size).

FIG. 8*a*.

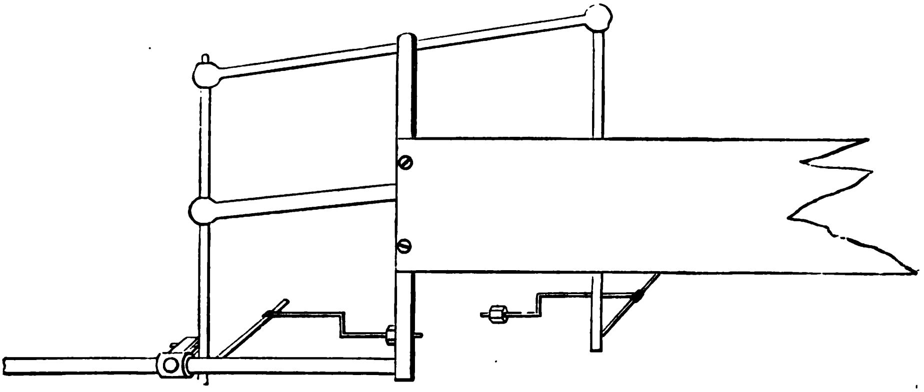


Wire frames depending like scale pans from ends of *bb*, Fig. 8, side and end views (half size).

Ultimately a small subsidiary beam, about 2·5 centims. long, was attached to the centre of the balance beam, just above the knife-edge (Fig. 8), the scale pans being represented by small wire frames in which the riders could rest (Fig. 8*a*). These frames depend from agate pieces resting on steel points at the extremities of the subsidiary beam in the way now usually adopted in delicate assay balances. This mode of supporting the riders appears to be perfectly satisfactory.

To raise or lower the riders two short horizontal lifting rods parallel to the beam move up and down within the supporting wire frames, with a nearly parallel motion, and on them are two metal pieces with their upper surfaces shaped so that the riders rest on them without swinging (Fig. 7, *r*). They are the extremities of **L**-shaped projections from a jointed parallelogram framework (Fig. 9), supported on an upright in front of the subsidiary beam. The framework is moved by a tongue engaging with it, and projecting from a horizontal rod, which rotates about its axis in bearings, one within the case and the other outside. The rod is turned through an angle of about 30° between stops by an endless string passing upwards and round a wheel in the observing room.

FIG. 9.



Lifting rods to raise or lower riders (half size).

The parallelogram framework and the bearing of the rotating rod within the case, are both supported independently of the case, from the ceiling. At first they were supported respectively on the central pillar of the balance, and on the case; but when the increase of optical sensitiveness enabled me to detect small irregularities, I realised how essential it was for accurate weighing, that all parts of the apparatus moved from the outside should be supported quite independently of the balance. Even the string moving the rod transmitted great and continual vibration. The rod and the framework with the lifting levers were, therefore, supported by iron rods coming down from the ceiling through holes in the top of the case, large pieces of cardboard stretching from these rods over the holes to hinder the passage of dust into the case. Once or twice in the course of preliminary experiments irregularities were traced to accidental contact of outside bodies with the case.

It appeared just possible that there might be electrification of the riders by friction with the lifting rods, especially when they were supported by cocoon silk. It was, therefore, advisable that the surface of the lifting rods should be of the same kind as that of the riders. As the latter are silver wire gilded, the lifting rods are also gilded. It may not be uninteresting to note here a curious phenomenon which occurred during some early preliminary experiments. The shaped pieces on the lifting rods were then of wood covered with gold leaf, put on with ordinary paste. After they had been on for some months, I obtained some very various results for the deflection due to the riders, and on examining the lifting rods, I found that a number of long needle growths projected from the wood pieces, and interfered with the supporting wire frames. At first I thought these were organic, but my colleague, Professor Hillhouse, examined them and found that they were crystalline. Doubtless, the hygroscopic paste set up electric action between the gold leaf and the brass, to which the wood pieces were attached, and the crystals were probably zinc sulphate. The wood was then replaced by brass gilded, and no further difficulty of the kind was experienced.

The length of the subsidiary beam was kindly determined for me by Mr. Glazebrook at the Cavendish Laboratory. The steel points are hardly sharp enough to determine the distance to 1 in 10,000, but the mean of the results is sufficiently exact. The following are Mr. Glazebrook's determinations; the four points being denoted by *a b c d*:—

Date.	Temperature.	Number of Readings.	*a* to *b*	Number of Readings.	*c* to *d*.
	°		Inches.		Inches.
1889 July 4 .	22·5	6	·9985	6	·9979
July 11 .	21·5	3	·9990	3	·9982
July 12 .	23	3	·9988	3	·9979

These are in terms of a gun-metal standard, of which the error is only 3 in 100,000 at 0°, and, therefore, for my purpose negligible. The beam is of brass, and we may assume with sufficient exactness that it has the same expansion as the standard. The temperature may, therefore, be left out of account. The mean value of $\frac{1}{2}(a\,b+c\,d)$ is therefore ·9983375 inch, or taking 2·539977 centims. to the inch we obtain

Length of beam at 0°, 2·53575 centims.

There is an advantage in fixing this beam at the centre, which should be noted here. Suppose the riders are not quite equal, but have values w and $w + \delta$. Let the two ends of the subsidiary beam be distant a and $a + l$ from the central knife-edge. Then the effect of picking up the rider w from the nearer, and letting down the rider $w + \delta$ on the further end, is equivalent to putting at unit distance

$$(w + \delta)(a + l) - wa = wl + \delta(a + l) = wl\left(1 + \frac{\delta}{w}\frac{a + l}{l}\right),$$

or the error δ / w is multiplied by $(a + l)/l$, and, if the beam is not central, $(a + l)/l$ may be greater than 1, so that the error is magnified.

If, however, the small beam is central, l is equal to $-2a$, and the error is multiplied by $+\frac{1}{2}$.

If the riders are interchanged and the weighings are then repeated, the mean result is the same as if riders with the mean value were used for

$$w(a + l) - (w + \delta)a = wl - \delta a = wl\left(1 - \frac{\delta}{w}\frac{a}{l}\right),$$

and the mean of this and the above is

$$\left(w + \frac{\delta}{2}\right)l.$$

The Attracting and Attracted Masses.—These are all made of an alloy of lead and antimony, for the sake of hardness, the specific gravity in each case being about 10·4. They were made at various times and places, the large attracting mass M being made more than twelve years ago by Messrs. Storey, of Manchester. The smaller balancing mass m was made in 1889 by Messrs. Heenan, of Manchester and Birmingham. These were both cast with a "head" on, and then turned. The attracted masses A and B were made by Messrs. Whitworth, and subjected to hydraulic pressure before turning. The dimensions have been measured from time to time, and there is no evidence of any sensible change of shape.

The larger mass M and the attracted masses A and B were weighed at the Mint through the kindness of the Deputy Master and Professor Roberts-Austen. For the weight of the balancing mass m, I am indebted to Messrs. Avery, of Birmingham. The large mass M has suffered two accidents since it was weighed, once being slightly cut into by a saw during some alteration of the case, and once being scratched by coming into contact with a piece of metal fixed to the turntable in taking it out of its place. The saw-cut was carefully filled in with lead, and the scratch removed only a fraction of a gramme, as was determined by taking a mould of the hollow. I should be glad to think that the determination of the attraction was sufficiently exact to make re-weighing necessary, but I am afraid that the alteration in weight is very far beyond the important figures, and I therefore take the original weight as sufficiently near the truth. The masses A and B have been gilded since the original weighing, but I carefully determined their increase of weight by the balance used in the gravitation experiment.

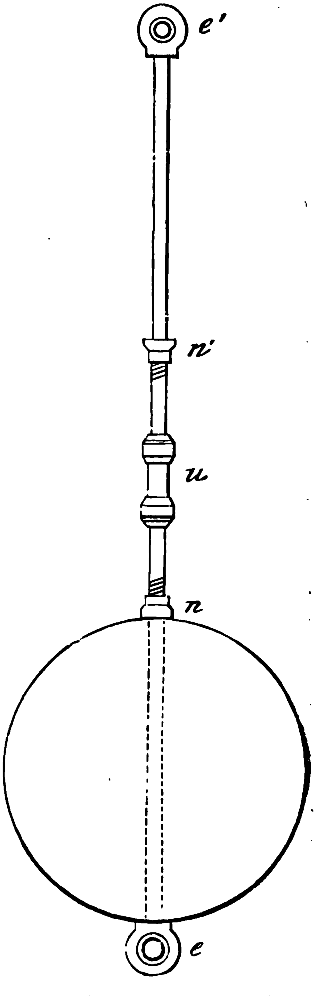
The values given below in the second column are the true masses. In the third column are the masses of M and *m*, less the air displaced by them, this being taken as 18·41 and 9·2 grms. respectively. It will be shown later that the true masses of A and B and the reduced masses of M and *m* may be used in the calculations of the result.

	True Mass in Grammes.	Mass less that of Air Displaced.
M	153407·26	153388·85
m	76497·4	76488·2
A	21582·33	
B	21566·21	

Suspension of the Attracted Masses.—Each of the attracted masses is drilled through along a diameter, the hole being ·6215 centims. in diameter, and a brass rod (Fig. 10) terminating in an eye *e* below, is passed through the hole. The mass is secured in position by a nut *n* working in a screw thread cut for a short distance in the rod. An exactly similar rod terminating in a similar eye *e′*, and with a similar nut *n′*, is fastened end to end to this by a union *u*. The nuts and the inner sides of the enlargements for the eyes are hollowed out so as to fit exactly on to the spheres.

From the ends of the balance beam hang down stout brass wires terminating in hooks. If these hooks are passed through the eyes *e′* the attracted masses are close to the floor of the balance case, and their centres are adjusted to be about 32 centims. from the centre of the large attracting mass when under either of them. If the masses are turned over so that the hooks pass through the eyes *e*, they are about 30 centims. higher or at nearly double the distance, the length *ee′* being about 48 centims. The rods being perfectly symmetrical about the union *u*, the attraction on them is the same in either position. The weight of each is about *212 grms., or* about $\frac{1}{100}$ of the attracted mass, so that any small variation in their position would produce a negligible variation in the total attraction. By the differential method, the attraction on them entirely disappears from the results.

FIG. 10.



Suspender for Attracted Mass (one-fourth size).

The Mode of Support of the Attracting Masses M **and** *m*.—This has already been described when describing the turntable.

The Riders.—Four centigrm. riders, A, B, C, D, of silver wire gilt were made by Mr. Oertling of the form shown in Fig. 7. These were weighed in 1886 at the Bureau International des Poids et Mesures, by M. Thiesen. The following is an extract from the certificate:—

"*Densité et Volume.*—Comme densité on a accepté celle de l'argent, et par conséquent comme volume de chacun des cavaliers, 0·0010 millilitre.

"*Détermination des poids des cavaliers.*—L'étude des poids de ces quatre cavaliers a été faite par M. le Dr. Thiesen, adjoint du Bureau International, chargé de la section des pesées. M. Thiesen au moyen de la balance Stüchrath, destinée à des poids au dessous du gramme, a d'abord déterminé les différences entre les quatre cavaliers pris deux à deux dans les six combinaisons possibles, et ensuite la différence entre l'ensemble des quatre cavaliers et le poids de 40 milligrms. de la série 0 du Bureau, série en platine iridié récemment étalonnée par M. Thiesen. Les comparaisons ont été faites du 19 au 29 Mars, 1886.

"*Résultats.*—De l'ensemble de ces comparaisons résultent les poids:—

A = 10·1247 milligrms.
B = 10·0615 „
C = 10·1196 „
D = 10·1262 „

"L'incertitude de ces déterminations ne dépasse pas 0·001 milligrm."

A and D were selected for use as being the nearest to each other in value. B and C were kept untouched in boxes till 1890. In the various experiments made between 1886 and the final weighings, A and D had necessarily been handled to some extent, especially through the frequent breaking of the silk fibre suspension used before the subsidiary beam described above, and it appeared possible that their weights might be altered. It was also necessary to determine whether an appreciable amount of dust was deposited on them in the course of several weeks as it was inconvenient to dust them frequently. The riders B and C might be assumed to have the same weight as in 1886, and could be taken as standards.

To make the weighings, a 16-inch chemical balance was arranged with a double suspension mirror on exactly the principle already described for the large balance. The apparatus was put together quickly with materials at hand, and might easily be greatly improved. It is only described here to show how accurate the method is, even with such rough apparatus, and that it is applicable to a small as well as a large balance.

A cork sliding on the pointer with a horizontal needle stuck in it, served to support one thread of the mirror; a stand with a projecting arm—one made to hold platinum wires in a Bunsen flame—served to support the other thread. A wire with a small copper vane depended from the mirror and was immersed in an oil dashpot. The telescope and a millimetre scale were on a level with the mirror about 2 metres distant on one side of the balance. Two brass strips, parallel to each other and the beam, were fixed on the top of one arm of the beam, and in each of these were two V notches in which centigramme riders could rest. Two levers, worked by cams on a rod rotated by the observer, picked one rider up and let down the other, so that the effect was equivalent to the transfer of 1 centigrm. from one notch to the other. Their distance apart was such that this was equivalent to the addition of ·3284 milligrm. to one pan of the balance. This was the arrangement described in my former paper. Attached to one pan was a pair of brass strips parallel to each other, and such that the riders A, B, C, or D, would just rest across them. Two lifting rods worked up and down between these strips, so that of the two riders to be compared, one could be picked up at the instant the other was let down. The lifting rods were worked by a rod rotated by the observer and supported quite independently of the balance and of the slab on which it rested. By this plan the value of the scale divisions and the shifting of the centre of swing on changing the weights to be compared, could all be determined without raising the beam of the balance between the successive weighings, an essential condition, I believe, for exact work.

The weighings were made in the large room of the Physical Laboratory, and no precaution was taken to protect the balance case beyond placing a board in front of it. The room is draughty and subject to great variations of temperature, so that the weighings were made under very disadvantageous circumstances. One result of this was a rapid and sometimes very great change of resting-point in the course of a few hours, so that the scale passed out of the field of view. In order to bring it back without opening the case, two glass tubes passed through the top of the case, almost down to the scale pans, and small bits of wire could be dropped through these on to either pan as needed. Caps fitted on to the tubes to prevent draughts. This plan appears worthy of mention, as it suggests a mode of determining the value of a scale division when a balance is either too sensitive for riders or has no special arrangement for their accurate use. If a piece of wire weighing, say, 1 milligrm. is cut into say ten nearly equal parts, and if these are dropped on to the two pans alternately the shiftings of the centre of swing will be to and fro, about equal distances, due to about ·1 milligrm., but the sum of the shiftings will be that due to 1 milligrm., and the balance at the end will be nearly in the same position as at the beginning.

The following is an abstract of the comparisons of the riders. They were made soon after the first determinations of attraction on February 4, when A and D had not been dusted for three months.

In each case three extremities of swing were observed, and the centre of swing was determined from these by the graphic construction described later (p. 101).

The centres of swing were combined in consecutive threes in the usual way to give the differences in scale divisions.

Thus, in the first series, the successive centres of swing with D and A alternately in the scale pan were

D	A	D	A	D
231	223	217	211·9	208

whence

$$(D - A)_1 = \frac{217 + 231}{2} - 223 = +\ 1{\cdot}0 \text{ division.}$$

$$(D - A)_2 = 217 - \frac{223 + 211{\cdot}9}{2} = -\ 0{\cdot}45 \text{ division.}$$

$$(D - A)_3 = \frac{217 + 208}{2} - 211{\cdot}9 = +\ 0{\cdot}6 \text{ division.}$$

$$\text{Mean } D - A = {\cdot}38 \text{ division.}$$

Successive values of the differences alone are given below. The time of swing one way was about 16 seconds.

February 16, 1890.

(1.) COMPARISON of A and D, undusted.

Deflection due to ·328 *milligrm.* 83·45, 82·45, 84·45 divisions. Mean 83·45 divisions.

D − A = 1·0, − ·45, + ·06 division. Mean ·38 division;

therefore D = A + ·0015 milligrm.

(2.) COMPARISON of A undusted, D dusted.

Value of scale division taken as in the last.

D − A = − ·5, + ·3, − ·1, − ·4, + ·25. Mean − ·09 division;

therefore D = A − ·0004 milligrm.

February 17, 1890.

(3.) COMPARISON of A and D, both dusted.

Value of scale division taken as below (4).

D − A = − ·1, − ·2, + ·3, − ·3, − ·8. Mean − ·22 division;

therefore D = A − ·0008 milligrm.

(4.) COMPARISON of C and D.

Deflection due to ·328 *milligrm.*, 85·35, 85·4, 84·65. Mean 85·13 divisions.

D − C = + ·15, ·00, + ·05, − ·15, + ·05, + ·3, + ·05, − ·05, − ·35, ·05, ·35, ·45, ·50, ·5. Mean ·114 division;

therefore D = C + ·00044 milligrm.

February 18, 1890.

(5.) COMPARISON of C and D repeated.

Deflection due to ·328 *milligrm.*, 92·75, 92·3, 91·65. Mean 92·23 divisions.

D − C = ·35, − ·05, − ·8, − ·95, − ·1, + ·05, 0, − ·15, − ·1, + ·05. Mean − ·17 division;

therefore D = C − ·0006 milligrm.

Combining this with the last, and weighting them in the ratio of the numbers of determinations in each,

D = C + (·00044 × 14 − ·0006 × 10) ÷ 24 = C − ·0000 milligrm.

(6.) COMPARISON of A and D.

Value of scale division taken as above, ·328 milligrm. = 92·23 divisions.

D − A = ·45, ·25, ·1, − ·2, − ·1, ·35, ·25, ·45, ·60, ·5, ·5, ·55, ·5, ·75, ·55, ·1, ·05, ·45, ·10, ·30, ·8, ·9, ·35, ·2, ·30, ·55, ·50, ·35, ·45, ·45.
Mean ·378 division;

therefore D = A + ·00134 milligrm.

Examining the values obtained in (1), (2), and (3), it will be seen that no trustworthy evidence is given of a difference due to dusting. Any existing difference was probably under ·002 milligrm., and since the weighings on February 4, before dusting, were made with the attracted masses in the upper position, when the attraction was only one-fourth of that on which the final results depend, we may safely neglect the effect. After this the riders were dusted more frequently, so that we may probably assume their values more constant.

The comparisons of C and D, and of A and D, in (4), (5), and (6), were made more carefully. That of A and D in (6) is much the best of the series, the air in the laboratory happening to be steadier while it was made. The range between the greatest and least values of the difference is one scale division, or ·0036 milligrm., and the different results are grouped fairly closely about the mean.

The centres of swing and the differences are plotted in Diagram VIII. I do not claim that these results show any remarkable accuracy when compared with those obtained at the Bureau International des Poids et Mesures, but remembering how rough the apparatus was, and how little precaution was taken to ward off air currents, I have not the slightest doubt that, with special design of apparatus and more suitable locality, the results could be very greatly improved, and the accuracy carried far beyond anything hitherto reached. As they stand, they seem to show the value of the combination of a short time of swing with optical magnification.

The result of comparisons (4), (5), and (6), is, that if C has its Paris value, viz., C = 10·1196 milligrms., then, A = 10·1183 milligrms., and D = 10·1196 milligrms.; whence $\frac{1}{2}(A + D) = 10{\cdot}119$ milligrms. This value may be used in calculating the result, since the riders were interchanged before Set II. was taken.

The losses experienced since 1886 by A and D are respectively, by A ·0064 milligrm., and by D ·0066 milligrm.—*i.e.*, they have diminished by practically equal amounts. This was to be expected, as they have probably received equal amounts of rough usage.

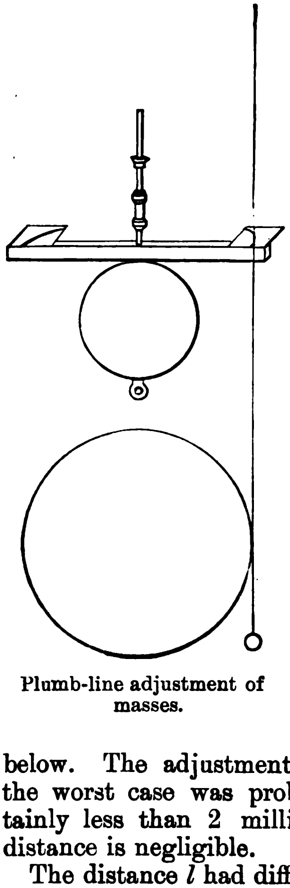
The substitution of the subsidiary beam for the cocoon fibre suspension of the riders having greatly diminished the handling to which they were subjected, I have not thought it necessary to weigh them again during the work.

LINEAR MEASUREMENTS.

In the mathematical theory it will be shown that the lengths required are those marked in Fig. 17, viz., the horizontal distances, L and l, and the vertical distances, D_1 D_2, d_1 d_2, H_1 H_2, h_1 h_2.

The Horizontal Distances.—Except when estimating the moment of the rider, the distance L is really that between the verticals through the centre of M and the centre of the more distant attracted mass. But the verticals through the centre of M in each position, so nearly passed through the centre of the mass above it, and, therefore, through the knife-edge from which it hung, that L was taken as equal to the length of the beam (p. 78).

FIG. 11.



Plumb-line adjustment of masses.

The accuracy of this adjustment was secured as follows. A horizontal cross-piece was fixed on the top of each attracted mass, with two horizontal cards at its two ends, each with a portion of a circular arc on it, with radius equal to that of the large mass M, and with centre over that of the attracted mass (Fig. 11). A plumb-line was then hung just in front of the case, and the balance was moved by the horizontal screws bearing against the base plate until the plumb-line always appeared to touch the circular arc above, when it appeared to touch the large mass below. The adjustment was not quite perfect, but the error in the worst case was probably not more than 1 millim., and certainly less than 2 millims. Such an error in the horizontal distance is negligible.

The distance l had different values for the two positions occupied by m on the turntable. Calling these values l_1 and l_2 respectively, $l_1 + l_2$ was found by measuring a, the inside distance between M and m, arranged as in Set II., and b, the inside distance between them, when m was put on the same side of the turntable as M, and adding to $a+b$ the sum of the diameters of M and m in the radial direction of the turntable as taken by square callipers.

The following are the values in terms of the cathetometer scale already referred to (p. 78), the temperature being 15° C.:—

$$\begin{aligned} a &= 157{\cdot}01 \\ b &= 33{\cdot}95 \\ \text{Diameter of M} &= 30{\cdot}52 \\ \text{,,} \quad \text{,,} \quad m &= 24{\cdot}23 \\ \hline \text{therefore} \qquad l_1 + l_2 &= 245{\cdot}71 \end{aligned}$$

The value of $l_1 - l_2$ was found by measuring the shortest distance of m from the wall when respectively in the first and second positions on the turntable. It was found that

$$\begin{aligned} l_1 - l_2 &= \quad {\cdot}12 \\ \text{whence} \qquad l_1 &= 122{\cdot}915 \\ l_2 &= 122{\cdot}795 \end{aligned}$$

We may obtain from these measures an independent value of the radius of the circle in which the centre of M moves. With perfect adjustment this should be $\frac{1}{2}$L = 61·66 at 18°.

It is equal to a + radius of M + radius of m − l_2, or, by the above measures,

$$\begin{aligned} &= 157{\cdot}01 + 15{\cdot}26 + 12{\cdot}115 - 122{\cdot}795 \\ &= 61{\cdot}59 \end{aligned}$$

which is only ·07 centim. less than $\frac{1}{2}$L.

Inasmuch as the wood probably expanded less than the cathetometer scale, while the metal expanded more, I have assumed as a rough approximation that the total expansion equalled that of the scale, so that the values of l_1 and l_2 are correct at 18°. No importance is, however, to be attached to this temperature correction.

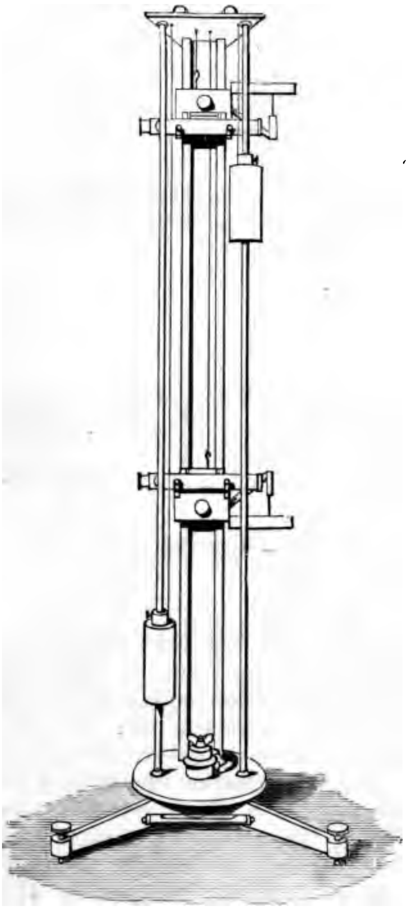
The Vertical Distances.—At the conclusion of each set of weighings with the attracted masses in a given position, the vertical distances between the top of the attracting masses and the bottom or top of the attracted masses (accordingly as they were in the upper or lower position) were measured by the cathetometer already referred to.

This instrument is of the well-known design of the Cambridge Scientific Instrument Company, and is especially adapted for measuring differences of level at different distances in different vertical planes. It reads to ·002 centim. The account of these measurements will be found in Table II. (p. 120 *et seq.*).

To find the distances D, d, H, h (Fig. 17), it was necessary to add to the actual distances measured the sum or difference of the vertical radii of the attracting and attracted masses, and, therefore, the vertical diameters of all the masses were measured.

For this purpose I used a cathetometer which has lately been constructed for me by Messrs. Bailey, of Bennett's Hill, Birmingham. I have to thank Mr. Potts, of that firm, for his care in its construction, and also for the trouble which he has taken in the construction and alteration of much of the apparatus used throughout the work recorded in this paper. As the cathetometer is, I believe, new in design and satisfactory in its performance, it appears worthy of description.

FIG. 12.



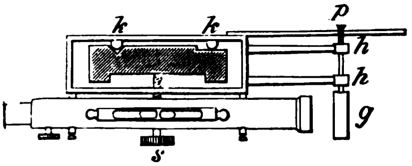
Cathetometer used to measure Vertical Diameters.

The Cathetometer used to measure Vertical Diameters (Fig. 12).—There are two telescopes, one to sight the upper, the other to sight the lower of the points between which the vertical height is required. There is no scale on the instrument, but after the telescopes are fixed to sight the two points the instrument is turned round a vertical axis, so that the telescopes sight a vertical scale at the same distance from them as the points. In general, of course, the cross wire will appear to lie between two divisions, but by means of the fine adjustment, to be described below, the two nearest scale divisions are brought in succession on to the cross wire, and by interpolation the reading corresponding to the point first sighted by the telescope is determined.

The telescopes are fixed on collars running up and down the main pillar, which has a section of the form shown in Fig. 13 (shaded).

The guides consist of three knobs, *k k*, on the inside of the collar, two sliding in a vertical V-groove and one on a plane, both groove and plane being at the back of the pillar. A screw, *s*, clamps the collar in any position. Gut strings running up over pulleys and supporting counterpoises, sliding on the thinner pillars (see Fig. 12), are attached to the collars so that these move easily. At first springs were used to keep the knobs always in contact, but I find it much better to remove these and trust merely to hand pressure to keep the collars in their proper position before clamping with the screw *s*.

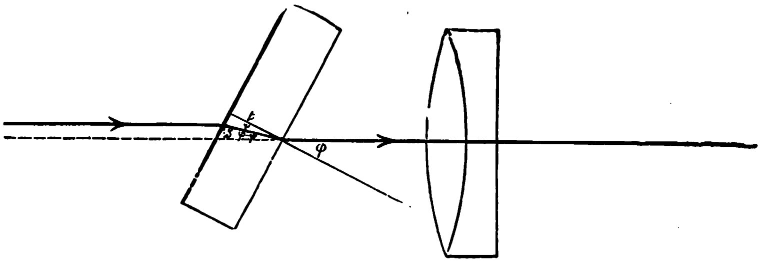
FIG. 13.



Section of pillar of new Cathetometer. *s*, clamping screw. *k*, *k*, guiding knobs. *g*, glass plate for fine adjustment, turning on axis *h*, *h*, with pointer at *p* perpendicular to plane of figure.

The fine adjustment is secured by the use of a piece of plate-glass placed in the front of each object glass* (*g*, Fig. 13), and capable of rotation about a horizontal axis, *h h*. A pointer is fixed on the end of this axis at *p*, and at its end is a small glass plate with a scratch on it moving close against a straight scale. If the plate is initially normal to the optic axis of the telescope, on turning it through ϕ, the ray which now comes along the optic axis has been shifted by transmission through the plate parallel to itself, a distance $t \sin(\phi - \psi) / \cos \psi$, where t is the thickness of the plate and ψ is the angle of refraction within it (see Fig. 14).

FIG. 14.



Section of fine adjustment plate.

This shifting happens for small angles to be nearly proportional to tan ϕ, and, therefore, to the reading on the straight scale. To

---

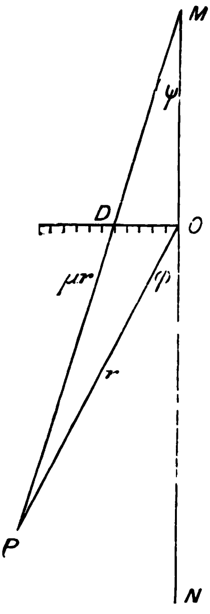
* Since this account of the adaptation of the parallel plate for fine adjustment was first published, I have found that it was used for micrometric work by Clausen in 1841, by Porro in 1842, and by Helmholtz in his Ophthalmometer. Porro used the plate both as an eyepiece micrometer and, as described in the text, in front of the object-glass ("Historical Note on the Parallel Plate Double-Image Micrometer," Monthly Notices R.A.S., liii. 5, p. 330).

show how nearly this is the case the following table gives the shifting for angles of 5°, 10°, and 20°, with a thickness of $t=1$ centim. and a refractive index $\mu=\frac{3}{2}$:

Angle ϕ =	Shifting.
5°	$\frac{1}{3}$ tan 5° (1 − ·00042)
10°	$\frac{1}{3}$ tan 10° (1 − ·00156)
20°	$\frac{1}{3}$ tan 20° (1 − ·0052)

The error in taking the shifting as proportional to tan ϕ is, up to 20°, quite negligible in ordinary telescope-cathetometer work. If it is desirable to have greater accuracy, it is probably best to use a table of corrections to the tangent; but it is possible to get an exact scale thus:

FIG. 15.



Let OP, Fig. 15, represent the pointer of length r, making ϕ with a line MN. Let a pointer PM jointed to this at P be of length μr, and let its extremity M move on the line MN. Drawing OD at right angles to MN, if s is the shifting, we have

$$\text{OD} = \text{OP}\,\frac{\sin(\phi-\psi)}{\cos\psi}$$

$$= \frac{rs}{t},$$

$$\therefore\ s = \frac{t}{r}\,\text{OD}.$$

Probably the practical difficulties in the use of such an arrangement would render it troublesome and uncertain.

The plate is used as follows: Adjust it normal to the optic axis of the telescope, and move the telescope till the required point is brought as near to the cross-wire as is possible by the hand. Clamp the telescope, and then turn the plate till the point is exactly on the cross-wire. Read the position of the pointer attached to the plate on its scale. Repeat these operations with the other telescope on the other point, then turn the instrument about its vertical axis till the telescopes sight the vertical scale placed at the same distance away as the two points. Looking through one of the telescopes the cross-wire is in general not exactly on a division. Turn the plate so that first the nearest division above, and next the nearest division below, is on the cross wire. Reading the position of the pointer in each case, interpolation gives us the reading on the vertical scale corresponding to the position of the pointer when the cross-wire was between the two scale divisions. Doing this for each telescope the difference between the two points is found in terms of the vertical scale.

The plates I have used are about 9 millims. thick, and the pointers about 9 centims. long. They move over scales such that 25 to 27 divisions correspond to a shifting of 1 millim. The lower scale is graduated from 0 to 50, the upper from 50 to 100 to prevent confusion. The 50 divisions occupy a distance of sixty-six millims.

It will be observed that in this form of instrument the level error is practically entirely obviated. It can only come in if the scale is not at the same distance as the height to be measured, and may then be made negligible in practice by levelling the telescopes. Indeed, the uncertainty of measurement appears only to depend on the uncertainty with which the cross-wire can be brought to the proper point, that is, it depends only on the magnifying power and definition of the telescopes used.

To illustrate the use of the instrument, a full account of the determinations of the vertical diameters is given in Table II. Below are the results, and for the sake of showing that there has certainly been no great change in shape, I give results obtained with a cathetometer more than ten years earlier at the Cavendish Laboratory at Cambridge.

—	1890.	1880.
	Centims.	Centims.
Large attracting mass M	30·526	30·5192
Small " " m	24·176	
Attracted mass A	15·8203	15·8166
" " B	15·7829	15·7842

The diameters of M and m in a horizontal direction parallel to a radius of the turntable measured by square callipers were

$$M = 30{\cdot}52 \text{ centims.}$$
$$m = 24{\cdot}23 \quad ,,$$

Temperature Correction.—Though the expansion of the masses was to be expected of an unimportant amount, I thought it advisable to attempt to measure it, in case there might be anything anomalous. One of the attracted spheres, B, was for this purpose placed between two vertical levers, in a tank through which could be run a continuous stream of cold or warm water. These levers depended from horizontal rods which could rock or slightly rotate on fine point suspensions. This was, in fact, a kind of double Lavoisier and Laplace apparatus. The motion of each lever was shown by another lever of about the same length, rising vertically up from each horizontal axis, and serving as the moving support for a double suspension mirror in which was viewed the reflection of a millimetre scale. Two telescopes and one scale were used for the two mirrors, though it would not have been difficult to arrange one telescope and two scales. The value of one scale division was determined by inserting a piece of thin glass between the sphere and each lever in turn. The method is exceedingly sensitive, but I have not been able to make it exact, owing to the warping produced in the rods due to unequal temperatures.

The measures of the expansion varied between ·0000214 and ·0000277, both vertical and horizontal diameters (in the position in the balance) being tested. The true value is probably nearly ·000025 or 1/40000. It will, therefore, lead to no appreciable error if we take the expansion as equal to that of the scale of the cathetometer, say 1/60000(see Table II.).

Determination of the Attraction by the Balance.

When the balance is used to measure such small forces and weights as those with which we are here concerned, it must be left swinging on its knife-edge throughout any set of weighings in which the deflections are to be compared one with another. For there is not the slightest reason to suppose that if the beam is lifted up and let down again, its new position of equilibrium will coincide with the old. And again, the beam, especially with such loads as the attracted masses, is put into a state of considerable strain, and continues to alter its shape sensibly for hours, and, probably, even days, after the masses are put on to it. I have, therefore, always left the beam free for at least two or three days before commencing work with the balance, and it has of course remained free during the course of each day's work. The balance room was never entered just before any weighing, as it took many hours for the disturbance due to entrance and interference with the case to die away.

When the turntable supporting the attracting masses is moved half round, from one stop to the other, the bulk of the attraction is taken away from one attracted mass and put on to the other. The balance, being free, is slightly tilted over to the side on which is the larger attracting mass. But the deflection in the apparatus as arranged is so very small—at the most only ten scale divisions—that errors of reading can only be neutralised by making a great number of successive measures.

Probably other errors are also largely eliminated, such as those due to the deposition of dust particles, shaking, change of ground level, and varying air currents. Of such errors I have found those due to varying air currents by far the worst. Sometimes—especially in autumn and winter—the balance will move quite irregularly through more than a scale division, and continue to move to and fro in this way for days or weeks. When in such an unsteady condition it is useless for accurate work. In spring and summer, however, it is much more steady as a rule, and frequently the scale can hardly be seen to move. I have never worked, when on looking into the telescope for some time the irregular movements appeared to be more than a fraction of a tenth—*i.e.*, a fraction of one of the diagonal divisions—though, doubtless, irregularities comparable with a tenth of a whole division have often made their appearance in the work. It is perhaps not safe to ascribe these always to air currents.

I have always found the air steadiest in warm quiet weather, with a slowly rising temperature in the balance room, and most unsteady after a sudden fall of temperature. As the alteration of temperature spreads downwards, this is fully in accord with Lord Rayleigh's observation that when the air is steady the ceiling is warmer than the floor, and that when it is unsteady the floor is the warmer of the two. In the observing room I had a gas stove often kept burning day and night, in the hope that the higher temperature it produced in the ceiling of the balance room below might steady the air. But the vertical walls of the balance room interfered with the action of the ceiling, and often produced unsteadiness.

A door opening or shutting anywhere in the building had a visible though transient effect, doubtless through an air wave. In a high wind the balance was always unsteady, partly, I suspect, through rushes of air into and out of the case with sudden pressure changes, and partly through changes of ground level, with variations of wind pressure against the building.

At all times there was a march in one direction or the other of the centre of swing. This was especially marked soon after the frame was lowered and the beam left free. As already remarked, readings were not taken till changes due to change in strain of the beam had subsided. But the march was very appreciable at other times, as will be seen from the diagrams. Perhaps the change was sometimes due to tilting of the ground, with barometric variation, since the balance was a very delicate level, and sometimes due to the change in buoyancy of the air affecting the two sides unequally, though I have not been able to make out any direct connection between barometric height and position of centre of swing. I believe that the explanation is to be sought for the most part in unsymmetrical effect on the beam of slight changes of temperature, for I have frequently noticed that a rising temperature produced an upward march, and a falling one a downward march. This explanation is supported by the following table of observations of the centre of swing, extending from May 9 to May 22, 1890, the balance being free, and the balance room undisturbed meanwhile.

The relation between temperature and the centre of swing is represented in diagram IX.

Date, 1890.	Time.	Centre of Swing.	Temperature.		Barometer.
			Balance Room.	Observing Room.	
			°	°	
May 9. .	11.5 A.M.	136·0	12·0	13.4	739·8
	12.55 P.M.	133·0	12·0	15·0	739·2
„ 12. .	11.15 A.M.	133·8	12·05	14·5	738·6
	1.15 P.M.	131·9	12·05	15·8	738·3
	2.40 P.M.	133·7	12·05	16·6	738·1
Stove left on all night of 12th–13th.					
„ 13. .	11.0 A.M.	181·7	12·6	17·5	740·2
	12.35 P.M.	181·5	12·6	18·4	740·3
	3.15 P.M.	185·0	12·7	18·6	740·3
Stove turned off.					
	5.25 P.M.	189·4	12·7	16·5	740·5
„ 14. .	11.20 A.M.	167·4	12·6	14·3	745·8
	1.10 P.M.	165·5	12·6	14·4	746·0
„ 15. .	11.5 A.M.	156·8	12·4	13·8	749·5
	2.45 P.M.	160·0	12·4	14·0	749·3
„ 16. .	1.25 P.M.	158·3	12·4	14·0	744·3
„ 17. .	8.50 P.M.	171·0	12·55	13·7	741·9
„ 19. .	10.30 A.M.	174·3	12·55	14·0	743·5
	6.5 P.M.	181·8	12·6	14·0	741·0
„ 20. .	11.30 A.M.	175·2	12·75	14·0	739·8
	1.5 P.M.	173·7	12·75	14·1	740·0
	5.20 P.M.	172·3	12·7	13·8	741·7
„ 21. .	11.10 A.M.	172·7	12·6	13·7	751·9
„ 22. .	11.30 A.M.	192 abt.	12·85	14·1	757·6

Of course, after a change in the position of the attracting masses or of the riders, the balance does not at once settle in a new position of equilibrium, but oscillates about it. Inasmuch as the balance never rests in this position, it is better to term it the centre of swing rather than the equilibrium position or resting point. The dashpot used to damp the vibrations of the mirror reflecting the scale serves also to damp those of the balance beam, and they die down rapidly. Instead of waiting, however, to observe directly the point on which they are closing in, it is much more exact, and also saves much time, to find the centre of swing as with an undamped balance from the extremities of the swings. I have always observed and recorded four extremities of three successive swings, occupying in all a little more than a minute.

Notwithstanding the very considerable damping, the successive lengths of swing are still in geometrical progression, but the rate of reduction is too great to allow the ordinary approximation, in which the geometrical is assumed to be an arithmetical progression. The exact method of determining the centre of swing is as follows:

Let a, b, c, d be four successive readings of extremities of swing and let x be the reading of the required centre.

Let the constant ratio of each swing length to the next be λ.

Then

$$a - x = \lambda (x - b) \qquad (1)$$

$$x - b = \lambda (c - x) \qquad (2)$$

$$c - x = \lambda (x - a) \qquad (3)$$

Eliminating λ from (1) and (2), we may readily obtain x in the form

$$x = c - \frac{(c - b)^2}{(a - b) + (c - b)} \qquad (4)$$

and from (2) and (3)

$$x = b + \frac{(c - b)^2}{(c - b) + (c - d)} \qquad (5)$$

With no disturbances and no errors of reading the values of x in (4) and (5) will coincide; but usually there is some small difference, the result of error or disturbance, and it is better to find both and take the mean. A third value might be obtained from (1) and (3); but it appears unadvisable to combine directly observations so far separated in time.

These formulæ lend themselves to easy arithmetical treatment, especially with the aid of a slide rule; but the following graphic method of finding the centre of swing is much less tiring and quite sufficiently exact.

Let the line OA, Fig. 16, represent the scale; O its zero, and A, B, C, D the points distant respectively a, b, c, d from O.

Let O′C′ be a parallel line, B′, C′, D′ being points opposite to B, C, D respectively. Let AB′ and BC′ intersect in K_1. Draw $X_1K_1X_1'$ perpendicular to OA. Then X_1 is the centre of swing given by equation (4). For

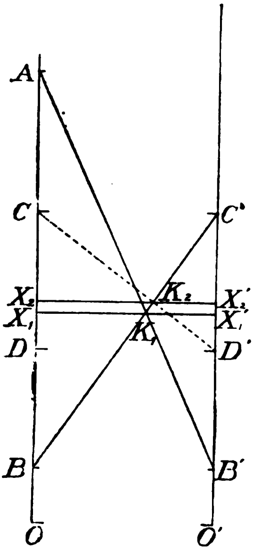
$$\frac{AX_1}{X_1B} = \frac{AX_1}{X_1'B'} = \frac{K_1X_1}{K_1X_1'} = \frac{X_1B}{X_1'C'} = \frac{X_1B}{X_1C},$$

i.e., X_1 is the point dividing AB and BC in the same ratio. Similarly if BC′ and CD′ intersect in K_2, and $X_2K_2X_2'$ be drawn perpendicular to OA, X_2 is the point given by equation (5).

The third point given by equations (1) and (3) is obtained from the intersection of AB′ and CD′, but evidently a small error in C or D′ may considerably alter the position of this point, and it is better not to use it.

The construction was carried out thus: a large opal glass plate, 10″ × 11″, was etched with cross lines 10 to the inch, so as to present the appearance of ordinary section paper. The glaze was taken off so that pencil marks could be made. A diagonal line ran at 45° across the plate through the corners of the inch squares, and this was always taken as the line BC′ in the figure. Taking any convenient horizontal line, usually, of course, far below the plate, as zero, each inch represented a scale division, each tenth a diagonal division. The values of b and c fixed the lines to be taken as OA, O′C, and on these were marked the points A, C, B′, D′. A long glass slip, with a straight scratch on it, was then laid across from A to B′ so that the scratch passed through A and B′, and its intersection K_1 with the diagonal BC′ was x_1 from the zero line. The slip was then laid with the scratch passing through C and D′, and its intersection K_2 with BC′ gave x_2. It will be observed that all the actual construction for a set of readings of the balance swings consisted in marking four points on the plate.

FIG. 16.



The following cases, the first of very regular, the second of very disturbed swing, will serve to compare the results by this exact method with those obtained from the ordinary arithmetic mean method. At the same time they will show how nearly constant is the ratio of swing decrease.

Date and Number, 1890.	Scale Readings in Diagonal Divisions.	Length of Swing.	Ratio of Each to Preceding.	Centre of Swing, Exact.	Centre of Swing, Approximate $\frac{a+2b+c}{4}$.
May 4, No. 3	865 939 896 921	74 43 25	·581 ·581	911·8 911·8	909·75 913·0
				Mean 911·8	Mean 911·375
Sept. 17, No. 45	1118 1053 1093 1068	65 40 25	·615 ·625	1077·8 1078·6	1079·25 1076·75
				Mean 1078·2	Mean 1078·0

In finding the attraction the observations were always made in the same order, the determination of the scale value of rider and attraction being sandwiched so that each might be equally affected by any comparatively slow changes. Starting with the initial position, the attracting masses and riders were so arranged that, on moving either, the balance was deflected in the same direction and over the same part of the scale.

The following was the order of proceeding always observed, the column headed "Centre of Swing" being supposed to contain the values of the position in each case determined from four swing extremities as just explained:—

	Centre of Swing.
(1) Initial position	i_1
(2) Riders moved	r_1
(3) Riders moved back to initial position	i_2
(4) Masses moved round	m_1
(5) Masses moved back to initial position	i_3
(6) Riders moved	r_2
(7) Riders moved back to initial position	i_4
(8) Masses moved round	m_2
and so on	

To minimise the effect of progressive changes these observations were always combined in threes in the following way. Denoting the scale value of rider by R, and of attraction by M:—

From (1), (2), (3) . . $R_1 = r_1 - \frac{i_1 + i_2}{2}$,

„ (3), (4), (5) . . $M_1 = m_1 - \frac{i_2 + i_3}{2}$,

„ (5), (6), (7) . . $R_2 = r_2 - \frac{i_3 + i_4}{2}$,

and so on.

These again were combined in threes, so that (the notation being continued) the successive values of attraction/rider are

$$\frac{2M_1}{R_1 + R_2}, \quad \frac{M_1 + M_2}{2\,\iota}, \quad \frac{2M_2}{R_2 + R_3}, \quad \text{and so on.}$$

The successive centres of swing i_1, r_1, i_2, m_1, &c., correspond to instants of time following each other at intervals of about 2 minutes, rather more than 1 minute being taken up in making and recording the four readings for each, and the rest in making the change of position in rider or mass and waiting for the next readings. It will be seen that each value of M or R is based on three successive centres of swing, the weighings extending over about 6 minutes, while each value of M/R is based on seven successive centres of swing determined in about 14 minutes.

A series of readings was usually continued for about 2 or 3 hours. The temperature in both observing and balance room was read at the beginning and end of the series, and the barometric height was also observed. As soon as possible after the desired number of determinations was completed with the attracted masses in one of the two positions, the vertical distances between attracting and attracted masses were measured by the cathetometer in the manner explained in Table II., and the position of the attracted masses was altered.

A full account of all the weighings is given in Table III., and the results are represented in Diagrams I.—VI. The three upper rows of points in each diagram represent the centres of swing, those in the initial position being marked ●. After movement of the rider they are marked ×, and after movement of the masses they are marked ○. The base lines for the different rows are altered to save space, as described on the diagrams, for on the scale adopted the rider series would always be about 10 inches above or below the initial series. In Diagram I. the rider and mass series are also brought down and superposed on the initial series, so that each of the three has the same average height. It will be seen that all three are affected by the same disturbances. The advantage of the short time of swing and the mode of combining the results in threes will be realised more easily from this superposition.

The base line may be regarded as a time scale, as the instants corresponding to successive centres of swing were almost exactly equidistant.

In each case, under the representation of the centres of swing, are plotted the resulting values of M/R and at the side will be found a representation of the distribution of results about the mean.

Assuming that each day's mean value is correct, and that the differences for different days are to be set down to variation of distance, &c., we can find the distribution of all the values about the mean by simply superposing the marginal curves at the side of the figures. The result fairly shows the accuracy as far as the weighing alone is concerned. It is represented in Diagram VII., where A is the mean value of the attraction in the lower, and a that in the upper position. A and a are brought near together to save space, but really they should be 40 inches apart. It will be seen that the range is about 2 per cent. of $A - a$ on each side of the mean, or taking the value of $A - a$ in milligrammes weight as about $\frac{1}{3}$ milligrm., and the load on each side as 20 kilogrms., the range is about $1/3 \times 10^9$. of this load on each side of the mean.

A comparison of the values of M/R in Diagrams I. and II., shows a very curious similarity in the fluctuations, and at first I was inclined to think there was some common external disturbance producing these fluctuations. But an analysis of the two sets of values appeared to show that the resemblance is merely accidental. When the values of M and R are set out separately, it is seen that the fluctuations depend chiefly on M, of which the fluctuations are slightly like each other for the two series, while those of R are quite different, but such that they make the fluctuations in M/R resemble each other much more closely than those in M alone. Further, it is not easy to see how fluctuations due to some external source would affect the values of M equally in the upper and lower positions and not have any effect on R. Some periodic change of level might be suspected, but this ought certainly to be traced in R. I have examined all the other diagrams and plotted out the component values of M and R, but have found no trace of resemblance, so that I think the curious likeness in I. and II. must be set down to accident.

There is a curious step by step descent of the centre of swing in the initial position on September 23, Diagram VI., which I cannot explain. It may be due to some error in the method of finding the centre of swing which comes in with a rapid march of that centre. The effect on the result is probably only small, for the value of M/R obtained with a march in the reverse direction on September 25 is very nearly the same, the two values being

September 23 . . . . ·2112753.
„ 25 . . . . ·2112533.

The following is a list of the weighings recorded, with the distances measured and the mean values of the attraction:—

Set I.

Date. 1890.	Position of Attracted Masses.	No. of Values of M/R.	Mean Values of M/R.	D or *d* in Centims.	H or *h* in Centims.
Feb. 4	Upper	50	·2142212	62·318	61·416
April 30 and May 4 .	Lower	100	1·0109685	31·783	30·824
May 25	Upper	50	·2157379	62·308	61·373

Set II.

Date. 1890.	Position of Attracted Masses.	No. of Values of M/R.	Mean Values of M/R.	D or *d* in Centims.	H or *h* in Centims.
July 28	Lower	25	·9973168	32·106	30·965
Sept. 17	Lower	25	·9984148	32·116	30·954
Sept. 23 and 25 . .	Upper	52	·2112647	62·708	61·566

On the completion of Set I. the four masses were inverted, and changed over from right to left or left to right, and the initial position was after this always arranged so that movement of rider or mass decreased the reading. This was done in order to lessen errors due to want of symmetry. If reversal had no effect, Set II. should, with the increased distance recorded above, give a value of M/R in the lower position of about ·990, instead of ·998. The larger value actually found is no doubt chiefly due to a want of symmetry in the large attracting mass M. The effect of this want of symmetry will be discussed after the investigation of the mathematical formula, and an account will be given of an independent method of detecting it. I think there is still outstanding a small difference, due, perhaps to want of symmetry in the turntable or in the attracted masses. The result of the reversal shows how necessary it was to make it. I should have liked to have in Set II. as many determinations as in Set I., so that the mean should be based on values of equal weight. During June and July 1890, a complete set of 100 in each position, upper and lower, was made; but, owing to the pressure of other work, I was unable to calculate the results till the completion of the set. I then found that the value of M/R was still more than in Set II., and, on plotting out the results, it appeared that occasionally the rider value fell very considerably, and in an irregular way. On examination, there was little doubt that the rider came in contact occasionally with the suspending frame, when it was raised and should have been clear from it. Very likely temperature changes had brought about a displacement of the lever apparatus. Comparison with Set I. seemed to show that during that set no such contact had taken place, for there was no comparable irregularity. As it appeared dangerous to attempt to disentangle the good from the bad, the set of June and July was rejected, and Set II. was taken as recorded. When I had made the weighings giving 50 and 52 values in the two positions respectively, the balance became so irregular, through the cooler weather, that it was useless to continue work. Rather than carry over the experiment into another season, when it might be necessary to repeat the whole of the work, I have preferred to take Set II. as it stands, and give it the same weight as Set I. The final results are calculated from the means of Sets I. and II., as explained hereafter. I may here state the results obtained:—

Constant of attraction . . $G = \frac{6{\cdot}6984}{10^8}$.

Mean density of the earth . $\Delta = 5{\cdot}4934$.

General Remarks on the Method.

Comparing the common balance with the torsion balance, there is no doubt that the former labours under the great disadvantage that the disturbances due to air currents are greatest in the vertical direction, that of the displacement to be measured. But even with this disadvantage the common balance may, I believe, be made to do much more than has hitherto been supposed possible. As an instrument in itself, apart from the external disturbances of air-currents, dust, &c., I believe its accuracy would be far beyond anything approached when these external disturbances are, as they always are, present to interfere with its action. I have always found that every precaution to ward off air currents and external disturbance has been accompanied by a corresponding increase in steadiness; and I have seen no sign of a limit of accuracy depending on the instrument itself.

Besides the protection from air currents, there are two conditions essential above all others for accurate work:—

1st. That during any set of weighings in which the deflections are to be compared with each other, the beam should be supported on its knife edge, and should be under constant strain.

2nd. That all moving parts, such as apparatus for changing riders or weights, should be supported quite independently of the balance or its case.

With regard to the first condition, it seems impossible to make the supporting frame move so truly and with so little disturbance that the knife edge shall return exactly to the same line. Even were it possible, the beam after raising and lowering would be practically a different beam, for, as my observations show, the condition of strain changes considerably after the load is first put on, and it would be merely a chance coincidence if the mean state of strain were the same during successive weighings. In the "Proceedings of the Royal Society," No. 190, 1878, one method of comparing weights of nearly equal value with the beam throughout on its knife-edge and equally strained is described,* and I should now only modify that method in having regard to the second condition, of which I have since realised the importance when working with the large balance and with increased optical sensitiveness. It is surprising to find how much disturbance is produced by having the moving parts of the apparatus connected with the balance or its case.

As to air currents there is no doubt that, as Professor Boys has shown, the greater the apparatus the greater the errors produced by them. At the time my apparatus was designed I did not know this, and there seemed to be a great advantage in making it large, as riders could be used of weight large enough to be measured accurately. Were I about to start with a new design I should certainly go towards the other extreme and make the apparatus

---

* I am glad that Dr. Thiesen urges the importance of this condition ("Travaux et Mémoires du Bureau International des Poids et Mesures,' vol. v., "Etudes sur la Balance").

small, attempting to get over the rider difficulty by some such method as that explained on p. 89. For not only is a smaller apparatus kept more easily at a uniform temperature, and, therefore, freer from the source of air currents, but it is much more handy to adjust, and even if the adjustments are not more accurate they will at least take much less time to make.

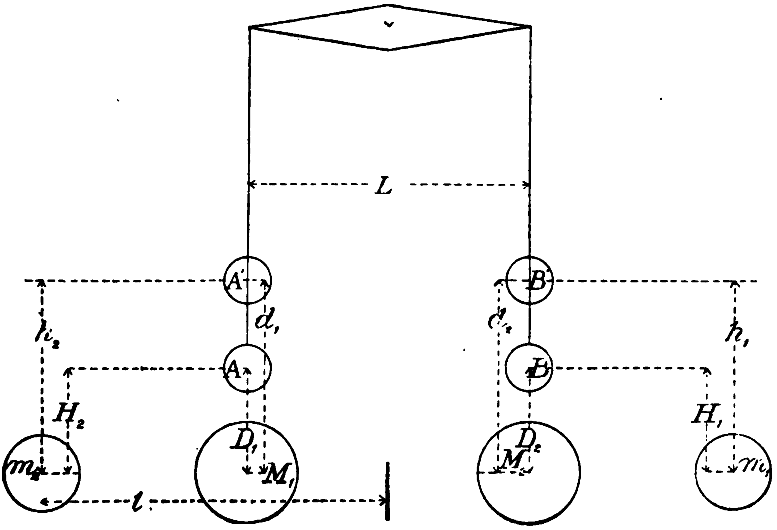
At the same time it is only fair to say, on behalf of the large apparatus, that some errors have been magnified on a like scale till they have become observable, and so could be investigated and eliminated. Starting with a small apparatus they would probably never have been detected, and would, therefore, have appeared in the final result.

II. MATHEMATICAL INVESTIGATION.

The Value of the Attraction Expressed in Terms of the Masses and Distances, and the Investigation of the Effect of Want of Symmetry in the Masses.

Let us suppose that initially the attracting masses are in the positions M_1, m_1, Fig. 17, the larger on the left, the smaller on the right, and that the attracted masses are in the lower position A, B. When the turntable is moved round so that the positions of the masses are M_2, m_2, the greater attraction is taken from the left and put on to the right. Let the centre of swing of the balance

FIG. 17.



alter by an amount corresponding to a total change of vertical pull of n dynes. Assuming that a spherical mass M attracts another spherical mass M′ when their centres are D centimetres apart with a force of GMM'/D^2 dynes, we can express the change of vertical pull due to the change of position of the masses as G × a function F of the masses and distances. There is also a change of pull on the suspending rods and the balance beam which we may denote by E.

Then $$n = GF + E.$$

In order to eliminate E let the attracted masses be moved into their upper positions A′, B′, and let the change on moving round the attracting masses be n' dynes. If f is the function of the masses and new distances corresponding to F,

$$n' = Gf + E.$$

Subtracting $$n - n' = G(F - f),$$

whence $$G = \frac{n - n}{F - f},$$

and knowing G, the mean density of the earth may be at once found in the manner shown later.

We have then to find the form of the functions, F, f, and as a preliminary step it is necessary to find the effect of the holes bored through the attracted masses A, B. This may be made to take the form of a correcting factor to the attraction which would be exercised on them if they were spheres.

The piece bored out in each case has radius ·31 centim. This we denote by c. It may be taken as practically a cylinder with plane ends and length equal to 15·8 centims., the diameter $2r$ of the spheres. The intensity due to such a cylinder of mass μ at D from its centre is (Todhunter's "An. Stat.," 5th ed., p. 293),

$$G\mu\,\frac{2r - \sqrt{\{(D + r)^2 + c^2\}} + \sqrt{\{(D - r)^2 + c^2\}}}{c^2 r},$$

which equals, to a sufficient approximation,

$$\frac{G\mu}{D^2 - r^2}.$$

If the mass remaining after μ is removed is A, and if the centre of the mass M is D below that of A, the attraction of M on A is

$$\frac{GM(A + \mu)}{D^2} - \frac{GM\mu}{D^2 - r^2}$$
$$= \frac{GMA}{D^2} + GM\mu\left(\frac{1}{D} - \frac{1}{D^2 - r^2}\right)$$
$$= \frac{GMA}{D^2}\left\{1 - \frac{\mu}{A}\left(\frac{r^2}{D^2} + \text{higher powers of } \frac{r^2}{D^2}\right)\right\}.$$

Now

$$\frac{\mu}{\mathrm{A}} = \frac{2\pi c^2 r}{\frac{4}{3}\pi r^2 - 2\pi c^2 r} = \frac{3}{2}\frac{c^2}{r^2} \text{ nearly } = \tfrac{3}{2}(\tfrac{31}{790})^2 = \cdot 00231,$$

and the greatest value of

$$\frac{r^2}{\mathrm{D}^2} = (\tfrac{79}{320})^2 = \cdot 061.$$

Then the higher powers may be neglected, and the attraction may be written

$$\frac{\mathrm{GMA}}{\mathrm{D}^2}\left(1 - \frac{3}{2}\frac{c^2}{\mathrm{D}^2}\right) = \frac{\mathrm{GMA}}{\mathrm{D}^2}(1 - \theta), \text{ say.}$$

When A and B are in the lower position, $\mathrm{D} = 32$, and $1 - \theta = \cdot 99986$. When they are in the upper position, $\mathrm{D} = 62$ and $1 - \theta = \cdot 99996$, a value so near 1 that we shall in this position omit the correction, since it is only applied to one-fourth of the final result.

In the cross attractions we shall also omit the correction.

Referring to Fig. 1 let the vertical differences of level between the centres of the various spheres be denoted as follows, the suffixes to M and m denoting their first and second positions respectively:—

$$\begin{array}{llll} \mathrm{A} - \mathrm{M}_1 = \mathrm{D}_1 & \qquad & \mathrm{B} - \mathrm{M}_1 = \mathrm{D}_1', \\ \mathrm{B} - \mathrm{M}_2 = \mathrm{D}_2 & & \mathrm{A} - \mathrm{M}_2 = \mathrm{D}_2', \\ \mathrm{B} - m_1 = \mathrm{H}_1 & & \mathrm{A} - m_1 = \mathrm{H}_1', \\ \mathrm{A} - m_2 = \mathrm{H}_2 & & \mathrm{B} - m_2 = \mathrm{H}_2'. \end{array}$$

When the masses A, B, are placed in their upper positions, let the corresponding distances be denoted by small letters.

Let the horizontal distance between the centres of A and B be L, being within sensible limits equal to that between the centres of M in its two positions, and to the length of the beam, and let the radius of the circle in which m moves be l.

Then we have the following horizontal distances:—

$$\begin{aligned} \mathrm{A} - \mathrm{M}_2 &= \mathrm{B} - \mathrm{M}_1 = \mathrm{L}, \\ \mathrm{A} - m_1 &= \mathrm{B} - m_2 = l + \tfrac{1}{2}\mathrm{L}, \\ \mathrm{A} - m_2 &= \mathrm{B} - m_1 = l - \tfrac{1}{2}\mathrm{L}. \end{aligned}$$

We may now write the change in vertical pull on the left by the motion of M from left to right, and of m from right to left, as follows—the first four terms representing the vertical attractions on A and B by M and m in their first position, the next four their attractions when moved round, and the last term E representing the change in attraction on the beam and suspending rods:—

$$G\left\{\frac{MA(1-\theta)}{D_1^2} - \frac{MBD_1'}{(D_1'^2+L^2)^{\frac{3}{2}}} - \frac{mBH_1}{\left\{H_1^2+\left(1-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}\right.$$
$$+\frac{mAH_1'}{\left\{H_1'^2+\left(l+\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} + \frac{MB(1-\theta)}{D_2^2} - \frac{MAD_2'}{(D_2'^2+L^2)^{\frac{3}{2}}}$$
$$\left.-\frac{mAH_2}{\left\{H_2^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} + \frac{mBH_2'}{\left\{H_2'^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}\right\} + E.$$

We may arrange all but the last term in nearly equal pairs.

Thus the first and fifth go together, and if we put $D_1 + D_2 = 2D$, and $D_1 + \delta = D_2 - \delta = D$, their sum is

$$GM(1-\theta)\left(\frac{A}{D_1^2}+\frac{B}{D_2^2}\right)$$
$$= GM(1-\theta\left\{\frac{A}{D^2}\left(1+\frac{2\delta}{D}+\frac{3\delta^2}{D^2}+\dots\right)\right.$$
$$\left.+\frac{B}{D^2}\left(1-\frac{2\delta}{D}+\frac{3\delta^2}{D^2}-\dots\right)\right\}$$
$$= GM(1-\theta)\frac{A+B}{D^2}\left\{\left(1+\frac{3\delta^2}{D^2}+\text{higher powers of }\frac{\delta^2}{D^2}\right)\right.$$
$$\left.+\frac{A-B}{A+B}\cdot\frac{\delta}{D}\left(2+\frac{4\delta^2}{D^2}+\dots\right)\right\}.$$

Now $(\delta/D)^2$ is negligible, as will be seen by reference to the table of distances, and $(A-B)/(A+B)$ is less than $4/3000$, or of the same order as δ/D.

To a sufficiently close approximation, then, the sum of the two terms is

$$\frac{GM(A+B)(1-\theta)}{D^2}.$$

The second and sixth terms may also be taken together, and putting

$$D_1' + D_2' = 2D' \text{ and } D_1' + \delta' = D_2' - \delta' = D',$$

we may show that to a sufficient approximation

$$GM\left\{\frac{BD_1'}{(D_1'^2+L^2)^{\frac{3}{2}}}+\frac{AD_2'}{(D_2'^2+L^2)^{\frac{3}{2}}}\right\} = \frac{GM(A+B)D'}{(D'^2+L^2)^{\frac{3}{2}}}.$$

The two pairs with m give similar results with

$$H = \tfrac{1}{2}(H_1 + H_2) \text{ and } H' = \tfrac{1}{2}(H_1' + H_2').$$

Now
$$\begin{aligned} 2D &= D_1 + D_2 = A - M_1 + B - M_2 \\ &= B - M_1 + A - M_2 = D_1' + D_2' = 2D', \end{aligned}$$

and similarly $2H = 2H'$, so that we may put the expression in the form

$$G\left\{\frac{M(A+B)(1-\theta)}{D^2} - \frac{M(A+B)D}{(D^2+L^2)^{\frac{3}{2}}} - \frac{m(A+B)H}{\left\{H^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} + \frac{m(A+B)H}{\left\{H^2+\left(l+\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}\right\} + E = GF + E \text{ say.}$$

It is evident that we may combine experiments at different distances on different occasions in the same way by taking D and H to represent the mean values of these distances, so long as there is only a small variation from the mean.

If the attracted masses are now moved into their upper positions the expression for the change in attraction may be at once deduced from that in the lower position by replacing D and H by d and h, and omitting the factor $1-\theta$. Let it be denoted by $Gf + E$.

Subtracting one expression from the other E is eliminated, and we have

$$\begin{aligned} G(F-f) = G\Bigg\{&\frac{M(A+B)(1-\theta)}{D^2} - \frac{M(A+B)D}{(D^2+L^2)^{\frac{3}{2}}} - \frac{m(A+B)H}{\left\{H^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} \\ &+ \frac{m(A+B)H}{\left\{H^2+\left(l+\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} - \frac{M(A+B)}{d^2} + \frac{M(A+B)d}{(d^2+L^2)^{\frac{3}{2}}} \\ &+ \frac{m(A+B)h}{\left\{h^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}} - \frac{m(A+B)h}{\left\{h^2+\left(l+\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}\Bigg\}. \end{aligned}$$

This is to be equated to the difference in the values of the change in attraction in the two positions, as determined by the rider.

Let

b = the length of the small rider beam.
w = the mass of each rider.
A = mass deflection ÷ rider deflection in lower position.
a = ,, ,, ,, ,, ,, upper ,,
g_B = acceleration of gravity, or dynes weight per unit mass at Birmingham.

Then
$$G(F-f) = \frac{(A-a)\,bwg}{\frac{1}{2}L}.$$

Whence we may find the gravitation constant

$$G = \frac{2bwg_{B}(A - a)}{L(-f)},$$

where all the quantities on the right hand are given in the Tables at the end.

The value of g_B may be found sufficiently nearly from the formula (Everett's "Units," p. 21); $g_B = 980{\cdot}6056 - 2{\cdot}5028 \cos 2\lambda - {\cdot}000003h$, where λ is the latitude $= 52^\circ\ 28'$ at Birmingham, and h is the height above sea-level, which may be taken as 450 feet, or 13,725 centims. Whence $g_B = 981{\cdot}21$.

Since all the operations are conducted in air, the effective masses should throughout be less by the mass of air each displaces. But since they all have nearly the same densities, and w and $A + B$ appear respectively in numerator and denominator, it is sufficient to take their true masses, and to correct for air displaced in the case of M and m only.

To obtain the mean density of the earth Δ, we must express the acceleration of gravity in terms of G and the mass and dimensions of the earth.

The ordinary formula (Pratt, "Figure of the Earth," 4th ed., p. 119) is based on the assumption that the earth is a spheroid. It is sufficiently correct for our purpose, the departure of the assumed spheroid from the actual shape being very small. Adding a term $-3 \times 10^{-9}h$, or approximately, -41×10^{-6}, since the balance room is taken as 13,725 centims. above sea-level (see above), the value of gravity at Birmingham may be written

$$g_B = \frac{GV\Delta}{a^2}\left\{1 + \frac{\varepsilon}{3} - \frac{3}{2}m + \left(\frac{5}{2}m - \varepsilon\right)\sin^2 52^\circ\ 28' - 41 \times 10^{-6}\right\},$$

where

V = volume of the earth = $1{\cdot}0832 \times 10^{27}$ (Everett's "Units," p. 57).
a = mean radius of the earth = $6{\cdot}3709 \times 10^{8}$ (*loc. cit.*).
Δ = mean density of the earth.
m = equatorial "centrifugal force" ÷ gravity = $\frac{1}{289}$.
ε = ellipticity of the earth = $\frac{1}{282}$.

The value of the ellipticity is taken to make the formula agree with that quoted above from Everett's "Units." The uncertainty in the value is quite unimportant, for were ε as low as $\frac{1}{295}$, the error in Δ, introduced by taking it as $\frac{1}{282}$, would be less than 1 in 50,000.

Substituting for G, the value of the mean density of the earth is

$$\Delta = \frac{a^2\mathrm{L}(\mathrm{F}-f)}{2bw\mathrm{V}\left\{1+\frac{\epsilon}{3}-\frac{3}{2}m+\left(\frac{5}{2}m-\epsilon\right)\sin^2 52^\circ\, 28' - 41\times 10^{-6}\right\}\left(\mathrm{A}-a\right)}.$$

Here, as in the value of G, w and A + B may have their true values, M and m their values less the mass of air displaced.

In the foregoing investigation we have supposed that all the masses are homogeneous and spherical, with the exception of the borings through A and B. We have supposed, also, that the turntable is exactly symmetrical about a vertical plane through its axis, so that its motion through two right angles is without effect. Doubtless these suppositions and the formula based on them are not quite true. But, if we invert all the masses and change their sides, or pervert the whole arrangement of them, on taking the mean of the results obtained in the original and inverted and perverted positions we ought to greatly reduce the errors. Indeed, those due to want of symmetry in the turntable should evidently be quite eliminated, and those due to want of homogeneity in the masses should certainly be lessened.

To show this, we shall calculate the effect of a spherical "blowhole," or gas cavity in M, in the first and most important term of F. This we shall take as being

$$\frac{\mathrm{GM}\,(\mathrm{A}+\mathrm{B})}{\mathrm{D}^2},$$

on the supposition that M is homogeneous and spherical.

If the mass of metal which would fill the blowhole is λ, supposing it to be placed there, the sphere is completed and its attraction is

$$\frac{\mathrm{G}\,(\mathrm{M}+\lambda)\,(\mathrm{A}+\mathrm{B})}{\mathrm{D}^2}.$$

But the vertical attraction is less than this in reality by the vertical component of the attraction of λ.

Let

B be the centre of the cavity,
P the centre of the attracted mass,
O the centre of the attracting mass,
δ the distance of B from the centre of M,
θ the angle BOP.

The vertical component of the attraction of λ is

$$\frac{\mathrm{G}\lambda\,(\mathrm{A}+\mathrm{B})\cos \mathrm{BPO}}{\mathrm{PB}^2},$$

but $\quad \mathrm{BP}^2 = \mathrm{D}^2 + \delta^2 - 2\mathrm{D}\delta\cos\theta,$

and
$$\cos \mathrm{BPO} = \frac{\mathrm{D} - \delta \cos\theta}{\mathrm{BP}},$$

whence the attraction of λ may be put

$$\frac{\mathrm{G}\lambda\,(\mathrm{A}+\mathrm{B})\,(\mathrm{D}-\delta\cos\theta)}{\mathrm{D}^2+\delta^2-2\mathrm{D}\delta\cos\theta^{\frac{3}{2}}} = -\mathrm{G}\lambda\,(\mathrm{A}+\mathrm{B})\,\frac{d}{d\mathrm{D}}\,\frac{1}{\sqrt{\mathrm{D}^2+\delta^2-2\mathrm{D}\delta\cos\theta}},$$
$$= \frac{\mathrm{G}\lambda\,(\mathrm{A}+\mathrm{B})}{\mathrm{D}^2}\left(1+2\mathrm{P}_1\,\frac{\delta}{\mathrm{D}}+3\mathrm{P}_2\,\frac{\delta^2}{\mathrm{D}^2}+\ldots\right),$$

where P_1, P_2 . . . are zonal harmonics. The attraction of the sphere with the cavity is therefore

$$\frac{\mathrm{GM}\,(\mathrm{A}+\mathrm{B}}{\mathrm{D}^2}\left\{1-\frac{\lambda}{\mathrm{M}}\left(2\mathrm{P}_1\,\frac{\delta}{\mathrm{D}}+3\mathrm{P}_2\,\frac{\delta^2}{\mathrm{D}^2}+4\mathrm{P}_3\,\frac{\delta^3}{\mathrm{D}^3}+\ldots\right)\right\}.$$

If the mass is inverted, the vertical component is obtained by changing the sign of δ, and the mean of the two values is

$$\frac{\mathrm{GM}\,(\mathrm{A}-\mathrm{B})}{\mathrm{D}^2}\left\{1+\frac{\lambda}{\mathrm{M}}\left(3\mathrm{P}_2\,\frac{\delta^2}{\mathrm{D}^2}+5\mathrm{P}_4\,\frac{\delta^4}{\mathrm{D}^4}+\ldots\right)\right\},$$

the first power of δ / D being eliminated.

If $\theta = 0$, P_2 and all the other harmonics $= 1$.

If $\theta = 90°$, $\mathrm{P}_2 = -\frac{1}{2}$, $\mathrm{P}_4 = \frac{3}{8}$, &c.

Now, with the actual dimensions of the apparatus, $(\delta/\mathrm{D})^2$ cannot be so great as $(\frac{1}{2})^2$ or $\frac{1}{4}$, and may, of course, be much smaller. The first term of those involving λ, therefore, is the most important, and it lies between $+\frac{3}{2}\,(\lambda/\mathrm{M})\,(\delta^2/\mathrm{D}^2)$ and $-3\,(\lambda/\mathrm{M})\,(\delta^2\mathrm{D}^2)$ changing sign for the value of θ given by $\mathrm{P}_2 = 0$.

The second set of experiments recorded in this paper was taken after inversion and change of side of all the masses, and the final result obtained from this set differs by a little more than 1 per cent. from that obtained from the first set, the observed attraction being slightly greater at the same distance. The difference may be due to irregularities in any or all of the masses and in the turntable, and to other undetected effects, such as change of level on rotating the turntable. It would be a very long task to disentangle these, and I have contented myself with trying to find how much must be set down to irregularity in the large mass M, by taking a set of weighings with it alone inverted.

After the weighings on July 28, and the subsequent measures of distances, M was inverted only, and the other masses remained as in Set II. Some weeks later on, September 14, 25 values of M/R = A were obtained, the mean being ·9926. The distances were D = 32·118, H = 30·978. The mass M was then put in its original position, as in Set II., and on September 17, on referring to the tables, it will be seen that the value of M/R obtained was ·9984, the distances being D = 32·117 and H = 30·955, practically the same as on September 14.

Assuming that the difference in attraction is due to cavities in various places, and that, for each, the term $3P_2\delta^2/D^2$ is negligible, we have, approximately,

$$\frac{1 - 2\dfrac{\Sigma\lambda P_1\delta}{MD}}{1 - 2\dfrac{\Sigma\lambda P_1\delta}{MD}} = \frac{9926}{9984}.$$

Whence, approximately, since D = 32.

$$\frac{\Sigma\lambda P_1\delta}{M} = \cdot 0464 \text{ centim.}$$

This result may be tested by independent experiment. For let the centre of gravity be x below the horizontal plane through the point bisecting the vertical diameter (*i.e.*, the centre of figure), in the position of September 14. The distance of any missing particle λ from the horizontal plane is $\delta \cos\theta = P_1\delta$. Completing the sphere by the addition of all such particles, the centre of gravity is brought to the centre of figure, so that we have

$$Mx = \Sigma\lambda P_1\delta,$$

and

$$x = \frac{\Sigma\lambda P_1\delta}{D}.$$

We have, therefore, to determine the vertical distance of the centre of gravity from the centre of figure.

In order to do this, a large flat-bottomed scale-pan (one belonging to the balance used in the gravitation experiment) was suspended by two parallel wires about 8 centims. apart and 3 metres long. In the middle of the pan was a shallow cup about 7·5 centims. internal diameter, arranged so that it could turn freely but truly about a vertical axis. The mass, M, was placed on this cup with the diameter, which had been vertical, arranged horizontal, and perpendicular to the plane of the suspending wires. A vertical flat plate, worked by a horizontal micrometer screw, could be brought just in contact with the end of the diameter, and the reading of the micrometer gave the position of the point of contact. The position of the scale-pan was determined by a plumb-line hanging over one edge in front of a horizontal scale. On turning the cup and mass through 180°, and repeating the readings, knowing the weight of the scale-pan and the position of its centre of gravity, x could at once be found.

Two separate experiments gave

$$x = \cdot 0536 \text{ centim.},$$

and $$x = \cdot 0516 \text{ centim.},$$

not very different from the value ·0464, obtained from the attraction experiments. The agreement is, I think, very close when it is noted that a difference in the attraction in one of the sets of weighings of 1 in 1000 would make x either ·038 or ·054.

This result appears to justify the rejection of all terms in the expansion above the first, and so supports the belief that the reversal largely eliminates errors due to irregularities of shape. For it is in the case of M that there is the greatest danger of large value for δ/D, and the above experiments seem to indicate that even in this case it is small.

It is, perhaps, noteworthy that the largest term rejected in the attraction of M, viz., $3\lambda P_2\delta^2/MD^2$ is, if we give P_2 its maximum value 1,

$$\frac{3\lambda\delta}{MD} \cdot \frac{\delta}{D} = \frac{3x}{D} \cdot \frac{\delta}{D},$$

which is not greater than

$$\frac{3 \times \cdot 0464}{32} \times \frac{15}{32} = \cdot 0020,$$

since the radius of the mass is 15.

This is in a term about 5/4 of the final result, so that the greatest error which can be introduced by neglecting this term is ·0025, or 1 in 400.

In calculating the results of the experiments the means of Sets I. and II. have been taken. Equal weights have been given to each set. It would have been more satisfactory if the number of experiments had been the same in each set; but I should have had to wait for another season to obtain more, and then it would, probably, have been necessary to repeat the whole series in both arrangements, as it is not safe to assume that the various disturbing causes remain the same over a wide interval of time. The second set, though fewer in number, are, in some respects, I believe, better; partly owing to the additional experience gained when they were taken.

In order that the various terms in $F - f$ may be compared, I give below their numerical values, as determined from the values of the masses and distances given in the tables. The meaning of each term in the first column will be seen on referring to Fig. 17. The second column contains the actual values; the third column the values in terms of the fourth, the lowest term.

VALUE of $F - f$.

$\frac{M(A+B)(1-\theta)}{D^2}$	= 6483938·8	416
$-\frac{M(A+B)D}{(D^2+L^2)^{\frac{3}{2}}}$	− 102416·3	6·6
$-\frac{m(A+B)H}{\left\{H^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}$	− 316243·3	20
$+\frac{m(A+B)H}{\left\{H^2+\left(l+\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}$	+ 15579·9	1
$-\frac{M(A+B)}{d^2}$	− 1693687·2	109
$+\frac{M(A+B)d}{(d^2+L^2)^{\frac{3}{2}}}$	+ 156728·0	10
$+\frac{m(A+B)h}{\left\{h^2+\left(l-\frac{L}{2}\right)^2\right\}^{\frac{3}{2}}}$	+ 310695·0	20
$-\frac{m(A+B)h}{\left\{h^2+\left(l+\frac{L}{2}\right)\right\}^{\frac{3}{2}}}$	− 27597·7	1·7

Whence $F - f = 4826997·2$.

The mean value of $A - a$ (see Table III.), is

$$A - a = ·791295;$$

substituting these values of $F - j$ and $A - a$ in the formula for G (p. 113), we obtain

$$G = \frac{6·6984}{10^8};$$

substituting them in the formula for Δ we obtain

$$\Delta = 5·4934.$$

The values given by Sets I. and II., treated separately, are to two figures of decimals:

Set I. $\Delta = 5·52$
Set II. $\Delta = 5·46$.

III. TABLES.

TABLE I.—Constants of the Apparatus and Dimensions of the Earth.

Masses.

		Grms.
Attracting mass M, in vacuo	=	153407·26
Less air displaced, say	=	153388·85
Attracting mass m, in vacuo	=	76497·4
Less air displaced, say	=	76488·2
Attracted mass A, in vacuo	=	21582·33
„ „ B, „	=	21566·21
„ „ A + B, in vacuo	=	43148·54
Riders each, in vacuo	=	0·010119

Vertical Diameters of Masses in Terms of Cathetometer Scale correct at 18°.

The masses are taken as having the same coefficient of expansion as the scale.

Centims.
M = 30·526
m = 24·176
A = 15·8203
B = 15·7829.

The diameters of the masses A and B are taken between the nuts securing them on the suspending wires.

		Centims.
Balance beam at 0°, L	=	123·232
Rider beam at 0°, b	=	2·53575
L/b (as occurring explicitly in G and Δ, independent of temperature, assuming them to have the same coefficient of expansion)	=	48·59775

Latitude of Birmingham	=	52° 28′
Height of balance room above sea-level	=	13725 centims.
Gravity at Birmingham, g_B	=	981·21 centims. /sec.^{2}
Mean radius of earth	=	6·3709 × 10^8 centims.
Volume of earth	=	1·0832 × 10^{27} c. centims
Equatorial " centrifugal force " / gravity	=	$\frac{1}{289}$
Ellipticity of earth	=	$\frac{1}{292}$

$$1 + \frac{\varepsilon}{3} - \frac{3m}{2} + \left(\frac{5}{2}m - 2\right)\sin^2 52^\circ 28' - 41 \times 10^{-6} \ldots\ldots = \cdot 999161.$$

TABLE II.—**Vertical and Horizontal Distances.**

Vertical Diameters of Masses taken by the Cathetometer (described p. 94).

In the tables below, P.S. signifies divisions on the scale over which moves the pointer, which is attached to the small adjustment plate. V.S. signifies divisions on the vertical millimetre scale.

DIAMETER of Large Attracting Mass M.

	Reading on pointer scale.	Mean.
Upper telescope sighting top of mass . .	73·2, 73·4, 73·2	73·27 P.S.
Lower telescope sighting bottom of mass .	23·0, 23·0, 23·2	23·07 P.S.

TURNING round to the Vertical Scale.

	Reading on pointer scale.	Mean.
Upper telescope sighting 459 millims. V.S.	94·6, 94·9, 94·4, 95·0, 94·0	94·58 P.S.
Upper telescope sighting 458 millims. V.S.	68·8, 68·8, 69·7, 69·0, 68·7, 70·0, 69·6, 69·4	69·25 P.S.

Therefore 25·33 P.S. divisions = 1 millim. V.S.,

and scale reading for top of mass $= 458 + \frac{73·27 - 69·25}{25·33}$

$= 458·158$ millims. V.S.

	Reading on pointer scale.	Mean.
Lower telescope sighting 153 millims. V.S.	27·3, 27·4, 27·7, 27·0	27·35 P.S.
Lower telescope sighting 152 millims. V.S.	0·0, 0·3, − 0·5, 0·0	− 0·07 P.S.

Therefore 27·42 P.S. divisions = 1 millim. V.S.,

and scale reading for bottom of mass $= 152 + \frac{23·07 + 0·07}{27·42}$

$= 152·844$ millims. V.S.

The difference = 30·5314 centims.

This is rather greater than the diameter of the mass, as the cross wire was made to touch the image of the mass in each case. A series of measures of 1 millim. on the scale, in which the cross wire was on the centre of each division, and of 1 millim. between the jaws of a wire gauge, in which the wire touched the images of the jaws, showed that at the distance at which the scale was, ·005 centim. must be subtracted, leaving

Diameter of M = 30·526 centims.

VERTICAL Diameter of Small Attracting Mass *m*.

	Reading on pointer scale.	Mean.
Upper telescope sighting top of mass . .	75·6, 75·6, 75·0	75·40 P.S.
Lower telescope sighting bottom of mass .	26·5, 26·3, 26·8	26·53 P.S.

TURNING round to the Vertical Scale.

	Reading on pointer scale.	Mean.
Upper telescope sighting 388 millims. V.S.	100, 99·9, 99·7	99·87 P.S.
Upper telescope sighting 387 millims. V.S.	73·9, 73·4, 74·0	73·77 P.S.

Therefore 26·10 P.S. divisions = 1 millim. V.S.,

$$\text{and scale reading for top of mass} = 387 + \frac{75·40 - 73·77}{26·10}$$

$$= 387·062 \text{ millims. V.S.}$$

	Reading on pointer scale.	Mean.
Lower telescope sighting 146 millims. V.S.	45·9, 45·9, 45·0	45·60 P.S.
Lower telescope sighting 145 millims. V.S.	20·4, 19·4, 20·0	19·93 P.S.

Therefore 25·77 P.S. divisions = 1 millim. V.S.,

$$\text{and scale reading for bottom of mass} = 145 + \frac{26·53 - 19·93}{25·77}$$

$$= 145·256 \text{ millims. V.S.}$$

The difference = 24·1806 centims.

Subtracting the same correction as in the last case for the cross wire,

Diameter of *m* = 24·176 centims.

Vertical Diameters of Attracted Masses A and B taken between the Junctions of the Securing Nuts with the Sphere.

A.

	Reading on pointer scale.	Mean.
Upper telescope sighting top of mass . .	82·7, 83·0, 82·9	82·9 P.S.
Lower telescope sighting bottom of mass .	31·0, 31·5, 31·3	31·3 P.S.

TURNING round to the Vertical Scale.

	Reading on pointer scale.	Mean.
Upper telescope sighting 429 millims. V.S.	95·0, 95·2, 95·3	95·3 P.S.
Upper telescope sighting 428 millims. V.S.	69·0, 69·0, 68·9	69·0 P.S.

Therefore 26·3 P.S. divisions = 1 millim. V.S.,

$$\text{and scale reading for top of mass} = 428 + \frac{82{\cdot}9 - 69{\cdot}0}{26{\cdot}3}$$

$$= 428{\cdot}531 \text{ millims. V.S.}$$

	Reading on pointer scale.	Mean.
Lower telescope sighting 271 millims. V.S.	48·0, 47·8, 48·4	48·1 P.S.
Lower telescope sighting 270 millims. V.S.	23·4, 23·0, 22·8	23·1 P.S.

Therefore 25·0 P.S. divisions = 1 millim. V.S.,

$$\text{and scale reading for bottom of mass} = 270 + \frac{31{\cdot}3 - 23{\cdot}1}{25{\cdot}0}$$

$$= 270{\cdot}328 \text{ millims. V.S.}$$

The difference gives the diameter since the middle of the cross wire was used, so that

Diameter of A = 15·8203 centims.

B.

	Reading on pointer scale.	Mean.
Upper telescope sighting top of mass . .	72·0, 71·0, 71·0	71·3 P.S.
Lower telescope sighting bottom of mass .	24·6, 25·0, 25·2	24·9 P.S.

TURNING round to the Vertical Scale.

	Reading on pointer scale.	Mean.
Upper telescope sighting 430 millims. V.S.	94·0, 94·6, 94·0	94·2 P.S.
Upper telescope sighting 429 millims. V.S.	68·0, 68·1, 68·1	68·1 P.S.

Therefore 26·1 P.S. divisions = 1 millim. V.S.,

$$\text{and scale reading for top of mass} = 429 + \frac{71{\cdot}3 - 68{\cdot}1}{26{\cdot}1}$$

$$= 429{\cdot}123 \text{ millims. V.S.}$$

	Reading on pointer scale.	Mean.
Lower telescope sighting 272 millims. V.S.	43·3, 43·0, 43·0	43·1 P.S.
Lower telescope sighting 271 millims. V.S.	17·1, 17·3, 17·4	17·3 P.S.

Therefore 25·8 P.S. divisions = 1 millim. V.S.,

$$\text{and scale reading for bottom of mass} = 271 + \frac{24{\cdot}9 - 17{\cdot}3}{25{\cdot}8}$$

$$= 271{\cdot}294 \text{ millims. V.S.}$$

And diameter of B = 15·7829 centims.

Vertical Distances between the Levels of the Centres of the Attracting and Attracted Masses Measured by Cathetometer.

The measurements were made as soon as possible after the completion of a set of weighings, usually on the following day.

It was necessary to fix the attracted masses in the position occupied during the weighings, and with the beam of the balance in the same strained condition. This was done in some cases, by gripping the left suspending wire by a pair of jaws; in others, by adding a small weight to one side, and placing a block of the right thickness under the mass on that side.

The cathetometer was placed in front of the left side of the balance case, from which position all the masses could be viewed by turning the telescope round the central pillar (Fig. 5). It was read when sighting the top of each attracting mass and the top of each attracted mass when in the lower position, the bottom of each when in the upper position, the top and bottom being taken at the junctions of the securing nuts with the masses. It is therefore necessary to add to the distances measured by the cathetometer the difference of the radii of attracting and attracted masses in the lower position, and their sum in the upper position (see p. 119). The work is shown in full for February 5 and May 5.

Tests were made at various times, showing that there was no change in the distances (at least within errors of reading), either through moving the turntable or in the course of a few days (see February 5 and May 5 for examples).

Temperature Correction.—The cathetometer scale is taken as correct at 18°, and its coefficient of expansion is assumed to be 1/60000. That of the masses is probably about 1/40000, but, for simplicity, is taken as equal to that of the scale, the difference, 1/120000, never amounting to as much as the errors of reading, since the greatest length concerned is 23 centims.

The temperature was estimated to be about 1° above that observed during the immediately preceding weighings, the presence of the observer and the lights used tending to raise it.

The cathetometer rested always on the brick floor of the room. Its vernier reads to ·002 centim.

SET I.

Attracted masses A on the left, B on the right. Attracting mass M moving round from left to right in front of the balance case.

February 5, 1890.—Attracted masses in upper position. Assumed temperature 11°.

Half way through the measurements the cathetometer was accidentally moved, and could not be exactly replaced. Repeating the reading of A it was found that ·197 must be added to the previous readings to compare with the following ones. This addition is made where the numbers have an asterisk.

	A.	B.	
	64·999* 65·001*	65·284 65·282	
m_2	M_1	M_2	m_1
23·448	25·895* 25·889*	26·070 26·064	23·947*
Differences: A − m_2 = 41·552	A − M_1 = 39·108	B − M_2 = 39·216	B − m_1 = 41·336

From Table I., p. 119, the sums of the radii of the masses are

$$\begin{aligned} R_M + R_A &= 23{\cdot}173, \\ R_M + R_B &= 23{\cdot}154, \\ R_m + R_A &= 19{\cdot}998, \\ R_m + R_B &= 19{\cdot}979. \end{aligned}$$

whence $d = \frac{1}{2}\{39{\cdot}108 + 23{\cdot}173 + 39{\cdot}216 + 23{\cdot}154\}$
$= 62{\cdot}326,$

and $h = \frac{1}{2}\{41{\cdot}336 + 19{\cdot}979 + 41{\cdot}552 + 19{\cdot}998\}$
$= 61{\cdot}433.$

These are in terms of a scale correct at 18°, so that the value is too great by about 7/60000. We take as true values

Corrected $d = 62{\cdot}318,$
$h = 61{\cdot}425.$

Test Experiment.—At the conclusion, the distance A − M_1 was measured again and found to be 39·110.

May 28, 1890.—Attracted masses in upper position. Assumed temperature 14°.

	A.	B.	
	64·674 64·674	65·286 65·288	
m_2	M_1	M_2	m_1
23·422 23·424	25·726 25·724	25·920 25·920	23·766 23·756
Differences: $A - m_2 = 41{\cdot}552$	$A - M_1 = 38{\cdot}949$	$B - M_2 = 39{\cdot}367$	$B - m_1 = 41{\cdot}526$

whence $d = 62{\cdot}312,$
$h = 61{\cdot}377.$

Subtracting temperature correction ·004,

Corrected $d = 62{\cdot}308,$
$h = 61{\cdot}373.$

Mean values in Set I., $d = 62{\cdot}313,$
$h = 61{\cdot}399.$

May 5, 1890.—Attracted masses in lower position. Assumed temperature 13°.

	A.	B.	
	50·324 50·324 50·328	50·622 50·634 50·630	
m_2	M_1	M_2	m_1
23·672 23·674	25·972 25·970 25·972	26·138 26·138 26·132	23·998 24·008
Differences: $A - m_2 = 26{\cdot}652$	$A - M_1 = 24{\cdot}354$	$B - M_2 = 24{\cdot}493$	$B - m_1 = 26{\cdot}626$

From Table I., p. 119,

$$R_M - R_A = 7{\cdot}353,$$
$$R_M - R_B = 7{\cdot}372,$$
$$R_m - R_A = 4{\cdot}178,$$
$$R_m - R_B = 4{\cdot}197,$$

whence $D = \frac{1}{2}\{24{\cdot}354 + 7{\cdot}353 + 24{\cdot}493 + 7{\cdot}372\}$
$= 31{\cdot}786,$

and $H = \frac{1}{2}\{26{\cdot}626 + 4{\cdot}197 + 26{\cdot}652 + 4{\cdot}178\}$
$= 30{\cdot}827.$

Subtracting temperature correction ·0025.
Corrected values for Set I.,

$$D = 31{\cdot}783,$$
$$H = 30{\cdot}824.$$

Test Experiment.—The balance was set free at the end of these measures, and two days later, on May 7, it was again fixed, and the distance D was determined by the cathetometer described on p. 94. The value obtained was D = 31·786.

NOTE.—If the apparatus were perfectly rigid and constant in its dimensions, we should expect $D - H = d - h =$ constant. The values actually given by the above experiment are

February 5	·892,
May 5	·959,
May 28	·935.

There is apparently a slight increase during the course of the spring, probably due to the warping of the wood supporting the mass *m*. But there was some uncertainty in sighting the top of the mass *m*, especially when in the position on the right.

SET II.

Attracted masses, A on the right, B on the left. Attracting mass M moving round from left to right behind the balance case. All the masses inverted.

July 29, 1890.—Attracted masses in lower position. Assumed temperature 16°.

	B.	A.	
	49·014	49·840	
	49·014	49·844	
m_1	M_2	M_1	m_2
22·434	24·584	24·788	22·868
22·436	24·586	24·782	22·864
Differences : $B - m_1 = 26·579$	$B - M_2 = 24·429$	$A - M_1 = 25·060$	$A - m_2 = 26·979$

whence D = 32·107,
H = 30·967.

Subtracting temperature correction ·001.

Corrected D = 32·106,
H = 30·966.

September 18, 1890.—Attracted masses in lower position. Assumed temperature 16°.

	B.	A.	
	49·076	49·768	
	49·074	49·766	
m_1	M_2	M_1	m_2
22·467	24·576	24·756	22·840
	24·576	24·758	
Differences : $B - m_1 = 26·608$	$B - M_2 = 24·499$	$A - M_1 = 25·010$	$A - m_2 = 26·927$

whence $D=82{\cdot}117$,
$H=30{\cdot}955$.

Subtracting temperature correction ·001,

Corrected $D=32{\cdot}116$,
$H=30{\cdot}954$.

Mean values in Set II., $D=32{\cdot}111$,
$H=30{\cdot}960$.

September 27, 1890.—Attracted masses in upper position. Assumed temperature 16°.

	B.	A.	
	63·880	64·540	
	63·876	64·544	
m_1	M_2	M_1	m_2
22·450	24·570	24·756	22·810
22·448	24·572	24·758	22·816
Differences : $B - m_1 = 41{\cdot}429$	$B - M_2 = 39{\cdot}307$	$A - M_1 = 39{\cdot}785$	$A - m_2 = 41{\cdot}729$

whence $d=62{\cdot}710$,
$h=61{\cdot}568$.

Subtracting temperature correction ·002,

Corrected values for Set II.,

$d=62{\cdot}708$,
$h=61{\cdot}566$,

NOTE.—The values of $D-H$ and and $d-h$, which should be constant, are from the above, and from another set of measures (not here recorded, see p. 115) on September 15, as follows. (We have no reason to expect the same value as in in Set I., as the masses M, *m*, have changed sides.)

July 29	1·140,
September 15	1·140,
September 18 . . , .	1·162,
September 27	1·142.

From July 29 to September 15 inclusive, the balance was swinging freely without alteration. The values of H should, therefore, be the same on those dates. They were

July 29	30·967
September 15 . . .	30·978

equal almost within errors of reading for the top of *m*.

Means of Sets I. and II.:

$$D = \tfrac{1}{2}(31{\cdot}783 + 32{\cdot}111)$$
$$= 31{\cdot}947.$$

$$H = \tfrac{1}{2}(30{\cdot}824 + 30{\cdot}960)$$
$$= 30{\cdot}892.$$

$$d = \tfrac{1}{2}(62{\cdot}313 + 62{\cdot}708)$$
$$= 62{\cdot}511.$$

$$h = \tfrac{1}{2}(61{\cdot}399 + 61{\cdot}566)$$
$$= 61{\cdot}483.$$

Horizontal Distances.

Set I.

$L = 123{\cdot}269$ centims.

At 18° $l_1 = 122{\cdot}915$ „

and $\frac{L}{2} = 61{\cdot}635$ „

Whence $l_1 + \frac{L}{2} = 184{\cdot}550$ „

$l_1 - \frac{L}{2} = 61{\cdot}280$ „

Taking the mean temperature of the Set as 12°, and assuming 1/60000 as the coefficient of expansion, on correcting to 12°,

$l_1 + \frac{L}{2} = 184{\cdot}532$ centims.

$l_1 - \frac{L}{2} = 61{\cdot}274$ „

Set II.

At 18° $l_2 = 122{\cdot}795$ centims.

$\frac{L}{2} = 61{\cdot}635$ „

Whence $l_2 + \frac{L}{2} = 184{\cdot}430$ „

$l_2 - \frac{L}{2} = 61{\cdot}160$ „

Taking the mean temperature of the Set as 15°, and correcting to 15°,

$$l_2 + \frac{L}{2} = 184{\cdot}421 \text{ centims.}$$

$$l_2 - \frac{L}{2} = 61{\cdot}157 \quad ,,$$

Mean values for the two Sets

$$L = 123{\cdot}260 \quad ,,$$

$$l + \frac{L}{2} = 184{\cdot}477 \quad ,,$$

$$l - \frac{L}{2} = 61{\cdot}216 \quad ,,$$

TABLE III.—**Determination of Attraction by the Balance.**

Determinations of Attraction in Terms of the Riders by the Balance.

In each case, four turning-points of three successive swings are recorded in tenths of a division—*i.e.*, in divisions on the diagonal lines. In the columns headed *i* the masses and riders are in the initial position, in those headed *r* the riders are moved, and in those headed *m* the masses are moved. Under each set of four readings is the calculated centre of swing (p. 101). In the next line are the deflections due to movements of riders and masses, each placed under the middle one of the three centres of swing from which it is calculated. In the next line are the values of deflection due to mass ÷ deflection due to rider, or M/R (p. 103).

SET I.

I. ATTRACTED Masses in Upper Position. Feb. 4, 1890, 7.59 P.M. to 10·49 P.M. Temperature: in observing room, 15°·7–16°·5; in balance room, 10°·05. Barometer 752·2–752·0 millims. Weather mild and still, after slight frost on the two previous nights. Time between successive passages of centre about 20 seconds.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings . .	725	912	725	804	764	913	726	804
	798	838	799	787	779	838	800	787
	759	878	759	796	771	879	759	797
	780	856	781	791	776	857	781	791
Centre of swing . .	772·55	863·85	773·00	792·80	773·90	864·60	773·40	793·20
Deflection due to rider or mass . .	—	91·075	—	19·350	—	90·950	—	19·700
Mass deflection ÷ rider deflection .	—	—	—	·212608	—	·214688	—	·217110

Table III.—(*continued*).

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings . .	763	913	724	804	764	914	725	805
	779	837	801	787	779	838	801	789
	771	879	759	796	771	880	760	796
	775	857	782	792	776	857	783	792
Centre of swing . .	773·60	864·25	773·85	793·05	773·90	865·05	774·50	793·65
Deflection due to rider or mass . .	—	90·525	—	19·175	—	90·850	—	18·950
Mass deflection ÷ rider deflection .	—	·214720	—	·211440	—	·209824	—	·208758

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings. . .	765	916	726	803	765	914	725	805
	780	839	803	788	779	838	800	787
	772	881	762	797	771	879	759	796
	777	859	784	792	776	857	782	791
Centre of swing . .	774·90	866·30	776·30	793·65	774·00	864·50	773·60	792·85
Deflection due to rider or mass . .	—	90·700	—	18·500	—	90·700	—	19·225
Mass deflection ÷ rider deflection .	—	·206174	—	·203966	—	·207966	—	·211438

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings . .	763	914	727	805	764	913	725	804
	780	838	800	789	779	838	800	786
	770	880	·760	797	771	879	760	796
	776	857	782	792	775	857	781	791
Centre of swing . .	773·65	865·05	774·15	793·90	773·65	864·65	773·80	792·50
Deflection due to rider or mass . .	—	91·150	—	20·000	—	90·925	—	19·275
Mass deflection ÷ rider deflection. .	—	·215167	—	·219690	—	·215975	—	·211494

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings . .	763	913	724	803	763	912	726	802
	778	838	801	789	778	837	799	787
	770	879	759	796	769	879	759	797
	774	857	782	792	774	857	780	792
Centre of Swing. .	772·65	864·60	773·85	793·50	772·30	864·20	772·95	793·30
Deflection due to rider or mass . .	—	91·350	—	20·425	—	91·575	—	19·875
Mass deflection ÷ rider deflection. .	—	·217296	—	·223316	—	·220038	—	·216503

TABLE III.—(*continued*).

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings . .	764 779 771 776	915 839 880 858	725 800 759 780	803 786 795 791	763 778 770 774	913 838 879 857	726 799 759 781	803 787 796 792
Centre of swing . .	773·90	865·55	773·15	792·25	772·70	864·60	773·15	792·95
Deflection due to rider or mass . .	—	92·025	—	19·325	—	91·675	—	19·200
Mass deflection ÷ rider deflection .	—	·212986	—	·210397	—	·210117	—	·209693

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)	*m.* (56)
Scale readings . .	764 779 772 776	914 839 880 858	724 802 759 781	803 787 795 790	762 778 769 774	912 836 877 855	721 798 755 779	801 785 794 788
Centre of swing . .	774·35	865·55	773·85	792·10	772·20	862·55	770·35	790·55
Deflection due to rider or mass . .	—	91·450	—	19·075	—	91·275	—	20·275
Mass deflection ÷ rider deflection .	—	·209267	—	·208784	—	·215557	—	·222161

	i. (57)	*r.* (58)	*i.* (59)	*m.* (60)	*i.* (61)	*r.* (62)	*i.* (63)*	*i.* (63*a*)	*m.* (64)
Scale readings	760 776 767 772	911 836 877 855	724 799 758 779	803 785 795 789	762 777 769 773	911 836 877 855	722 800 758 780	725 799 759 780	803 786 796 790
Centre of swing	770·20	862·50	772·20	791·35	771·70	862·50	772·50	772·90	792·30
Deflection due to rider or mass . . .	—	91·250	—	19·425	—	90·400	—	—	19·900
Mass deflection ÷ rider deflection .	—	·217534	—	·213873	—	·217506	—	—	·217873

* After 63 the riders were moved by mistake instead of the masses, therefore it was necessary to return to the initial position, and take the readings in (63*a*).

TABLE III.—(*continued*).

	i. (65)	*r.* (6	*i.* (67)	*m.* (68)	*i.* (69)	*r.* (70)	*i.* (71)	*m.* (72)
Scale readings . .	762 777 769 774	913 838 879 857	725 800 758 780	802 785 794 789	759 776 767 772	909 834 875 854	722 797 757 779	802 784 794 789
Centre of swing . .	771·90	864·60	772·75	790·80	770·15	860·80	771·05	790·55
Deflection due to rider or mass . .	—	92·275	—	19·350	—	90·200	—	19·450
Mass deflection ÷ rider deflection .	—	·213170	—	·212084	—	·215078	—	·214947

	i. (73)	*r.* (74)	*i.* (75)	*m.* (76)	*i.* (77)	*r.* (78)	*i.* (79)	*m.* (80)
Scale readings . -	760 777 768 773	911 835 877 854	724 798 757 779	803 785 795 790	762 777 769 773	911 835 877 854	721 797 756 778	800 784 793 788
Centre of swing . .	771·15	862·05	771·40	791·50	771·70	862·05	770·30	789·75
Deflection due to rider or mass . .	—	90·775	—	19·950	—	91·050	—	20·150
Mass deflection ÷ rider deflection .	—	·217020	—	·219442	—	·220209	—	·221064

	i. (81)	*r.* (82)	*i.* (83)	*m.* (84)	*i.* (85)	*r.* (86)	*i.* (87)	*m.* (88)
Scale readings . .	759 774 766 771	910 833 876 854	722 796 757 778	801 783 793 787	759 775 767 771	910 834 876 854	723 797 757 778	802 783 793 787
Centre of swing . .	768·90	860·95	770·50	789·30	769·60	861·25	770·90	789·35
Deflection due to rider or mass . .	—	91·250	—	19·250	—	91·000	—	19·300
Mass deflection ÷ rider deflection .	—	·215990	—	·211248	—	·211813	—	·212995

	i. (89)	*r.* (90)	*i.* (91)	*m.* (92)	*i.* (93)	*r.* (94)	*i.* (95)	*m.* (96)
Scale readings . .	759 775 766 771	908 831 874 852	719 795 754 776	800 783 792 788	760 775 767 772	910 835 876 854	723 798 756 779	800 785 793 789
Centre of swing . .	769·20	859·00	768·35	789·05	769·90	861·60	770·90	790·30
Deflection due to rider or mass . .	—	90·225	—	19·925	—	91·200	—	19·350
Mass deflection ÷ rider deflection .	—	·217373	—	·219650	—	·215323	—	·212462

TABLE III.—(*continued*).

	i. (97)	*r.* (98)	*i.* (99)	*m.* (100)	*i.* (101)	*r.* (102)	*i.* (103)	*m.* (104)	*i.* (105)
Scale readings	761 777 768 772	910 835 876 853	721 796 756 777	798 783 791 787	759 775 765 770	909 833 874 852	721 796 756 777	800 783 793 787	759 775 767 771
Centre of swing	771·00	861·35	769·80	788·35	768·60	859·65	769·80	789·35	769·60
Deflection due to rider or mass . . .	—	90·950	—	19·150	—	90·450	—	19·650	—
Mass deflection ÷ rider deflection .	—	·211655	—	·211136	—	·214483	—	—	—

Feb. 4, 1890. Mean of 50 determinations of M/R = *a* }
Attracted masses in upper position } ·21422122.

II. ATTRACTED Masses in Lower Position. April 30, 1890, 7.45 P.M. to 10.32 P.M. Temperature: in observing room, 17°–16°·1; in balance room, 11°·1; barometer, 748·6–749·2 millims. Weather clear; S.E. wind; sunny during day. Time between successive passages of centre not quite 20 seconds.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings . .	1046 969 1012 988	1133 1055 1098 1075	951 1024 984 1007	1127 1062 1099 1078	955 1025 986 1007	1134 1059 1102 1077	952 1028 985 1009	1123 1069 1099 1082
Centre of swing . .	996·60	1082·85	998·35	1085·50	999·80	1086·25	1000·40	1088·25
Deflection due to rider or mass .	—	85·375	—	86·425	—	86·150	—	87·350
Mass deflection ÷ rider deflection .	—	—	—	1·00772	—	1·00856	—	1·01437

TABLE III.—(*continued*).

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings . .	962	1136	955	1129	961	1136	956	1133
	1023	1060	1029	1067	1027	1064	1031	1069
	989	1104	987	1102	989	1105	989	1103
	1009	1079	1012	1083	1012	1081	1013	1085
Centre of swing . .	1001·40	1088·00	1002·45	1089·55	1003·15	1090·00	1004·15	1091·25
Deflection due to rider or mass .	—	86·075	—	86·750	—	86·350	—	86·350
Mass deflection ÷ rider deflection .	—	1·01133	—	1·00623	—	1·00232	—	1·00101

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings . .	967	1141	957	1135	965	1143	958	1134
	1027	1064	1034	1070	1031	1066	1036	1073
	994	1108	990	1106	994	1110	993	1108
	1012	1083	1015	1086	1015	1085	1017	1089
Centre of swing . .	1005·65	1092·00	1006·00	1093·20	1007·40	1094·00	1008·35	1095·40
Deflection due to rider or mass .	—	86·175	—	86·500	—	86·125	—	86·825
Mass deflection ÷ rider deflection.	—	1·00290	—	1·00406	—	1·00624	—	1·01106

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings . .	976	1143	963	1141	966	1145	960	1141
	1027	1069	1037	1073	1037	1070	1040	1075
	998	1110	996	1112	996	1112	995	1113
	1016	1087	1019	1090	1019	1089	1020	1092
Centre of swing . .	1008·80	1095·35	1010·65	1097·90	1010·85	1097·05	1011·15	1099·30
Deflection due to rider or mass .	—	85·625	—	87·150	—	86·050	—	87·575
Mass deflection ÷ rider deflection .	—	1·01591	—	1·01529	—	1·01264	—	1·01713

TABLE III.—(*continued*).

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings . .	973 1034 1000 1020	1147 1071 1114 1090	962 1041 996 1022	1137 1079 1112 1094	975 1034 1002 1020	1146 1075 1114 1091	965 1042 999 1021	1138 1080 1113 1094
Centre of swing . .	1012·30	1098·55	1012·50	1100·20	1013·40	1099·80	1014·00	1101·00
Deflection due to rider or mass. .	—	86·150	—	87·250	—	86·100	—	86·650
Mass deflection ÷ rider deflection.	—	1·01465	—	1·01306	—	1·00987	—	1·00858

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings . .	977 1035 1003 1022	1150 1073 1116 1093	964 1043 1000 1025	1089 1110 1098 1104	976 1038 1004 1023	1153 1074 1118 1094	968 1044 1001 1025	1148 1078 1118 1095
Centre of swing . .	1014·70	1100·80	1015·45	1102·20	1016·15	1102·35	1016·45	1103·40
Deflection due to rider or mass. .	—	85·725	—	86·400	—	86·050	—	86·475
Mass deflection ÷ rider deflection.	—	1·00933	—	1·00597	—	1·00450	—	1·00625

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)	*m.* (56)
Scale readings . .	978 1039 1005 1025	1153 1076 1119 1094	969 1045 1002 1027	1149 1080 1118 1097	976 1043 1005 1026	1153 1079 1121 1096	968 1047 1004 1028	1145 1085 1118 1099
Centre of swing . .	1017·40	1103·35	1017·65	1104·45	1018·60	1105·55	1019·25	1106·15
Deflection due to rider or mass .	—	85·825	—	86·325	—	86·625	—	86·875
Mass deflection ÷ rider deflection.	—	1·00670	—	1·00116	—	·99971	—	1·00973

Table III.—(*continued*).

	i. (57)	*r.* (58)	*i.* (59)	*m.* (60)	*i.* (61)	*r.* (62)	*i.* (63)	*m.* (64)
Scale readings . .	984 1039 1008 1026	1155 1078 1120 1097	971 1048 1004 1029	1151 1083 1122 1100	977 1046 1007 1030	1157 1081 1123 1100	972 1051 1007 1031	1152 1087 1123 1102
Centre of swing . .	1019·30	1105·10	1020·00	1107·85	1021·25	1108·10	1022·60	1109·95
Deflection due to rider or mass .	—	85·450	—	87·225	—	86·175	—	86·850
Mass deflection ÷ rider deflection.	—	1·01872	—	1·01646	—	1·01011	—	1·00798

	i. (65)	*r.* (66)	*i.* (67)	*m.* (68)	*i.* (69)	*r.* (70)	*i.* (71)	*m.* (72)
Scale readings . .	983 1046 1011 1031	1159 1082 1125 1102	976 1051 1008 1033	1153 1088 1126 1104	983 1049 1012 1032	1161 1083 1127 1102	978 1053 1011 1034	1158 1088 1127 1106
Centre of swing . .	1023·60	1109·80	1023·70	1112·05	1025·15	1111·05	1025·95	1113·15
Deflection due to rider or mass .	—	86·150	—	87·625	—	85·500	—	86·700
Mass deflection ÷ rider deflection.	—	1·01262	—	1·02097	—	1·01944	—	1·01226

	i. (73)	*r.* (74)	*i.* (75)	*m.* (76)	*i.* (77)	*r.* (78)	*i.* (79)	*m.* (80)
Scale readings . .	985 1051 1014 1033	1163 1086 1129 1104	980 1056 1013 1036	1156 1092 1129 1108	990 1051 1017 1036	1163 1087 1131 1107	982 1057 1015 1039	1161 1093 1132 1109
Centre of swing . .	1026·95	1113·40	1028·25	1115·50	1029·20	1115·20	1030·15	1117·65
Deflection due to rider or mass .	—	85·880	—	86·775	—	85·525	—	87·100
Mass deflection ÷ rider deflection .	—	1·01093	—	1·01299	—	1·01652	—	1·01782

TABLE III.—(*continued*).

	i. (81)	*r.* (82)	*i.* (83)	*m.* (84)	*i.* (85)	*r.* (86)	*i.* (87)	*m.* (88)
Scale readings . .	991 1054 1018 1038	1167 1090 1133 1108	984 1059 1018 1041	1158 1098 1132 1112	992 1056 1021 1041	1169 1092 1136 1111	984 1063 1018 1042	1161 1100 1135 1115
Centre of swing . .	1030·95	1117·40	1032·60	1119·55	1033·55	1120·00	1034·00	1122·20
Deflection due to rider or mass .	—	85·625	—	86·475	—	86·225	—	87·600
Mass deflection ÷ rider deflection.	—	1·01358	—	1·00640	—	1·00942	—	1·01890

	i. (89)	*r.* (90)	*i.* (91)	*m.* (92)	*i.* (93)	*r.* (94)	*i.* (95)	*m.* (96)
Scale readings . .	996 1058 1022 1043	1171 1094 1137 1114	987 1064 1022 1045	1165 1100 1137 1117	995 1061 1024 1045	1172 1097 1137 1116	989 1066 1023 1046	1169 1099 1140 1117
Centre of swing . .	1035·20	1121·75	1036·85	1123·75	1037·35	1123·15	1038·20	1125·05
Deflection due to rider or mass .	—	85·725	—	86·650	—	85·375	—	86·575
Mass deflection ÷ rider deflection.	—	1·01633	—	1·01286	—	1·01450	—	1·01065

	i. (97)	*r.* (98)	*i.* (99)	*m* (100)	*i.* (101)	*r.* (102)	*i.* (103)	*m.* (104)	*i.* (105)
Scale readings . .	998 1062 1026 1045	1175 1097 1141 1116	992 1066 1025 1047	1174 1102 1141 1119	995 1064 1027 1048	1176 1098 1148 1118	995 1067 1026 1049	1169 1105 1143 1121	1001 1066 1029 1049
Centre of swing . .	1038·75	1125·05	1039·45	1127·15	1040·15	1126·70	1040·75	1128·90	1042·20
Deflection due to rider or mass .	—	85·950	—	87·350	—	86·250	—	87·425	
Mass deflection ÷ rider deflection.	—	1·01178	—	1·01452	—	1·01319	—	—	

April 30.	Mean of 50 determinations of M/R = A Attracted masses in lower position	1·010905.

TABLE III.—(*continued*).

MAY 4, 1890, 11.11 to 11.50 A.M. Temperature: in observing room, 13°·5–13°·8; in balance room, 11°·7; barometer, 742·0–741·7 millims. Weather inclined to rain; a little cooler than previous day; wind S. to S.W.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings . .	875 936 900 920	1045 969 1013 988	865 939 896 921	1044 970 1014 989	865 938 897 920	1045 967 1013 986	861 940 894 921	1041 971 1012 989
Centre of swing . .	913·10	996·95	911·80	997·75	911·60	996·00	910·95	997·10
Deflection due to rider or mass . .	—	84·50	—	86·050	—	84·725	—	86·275
Mass deflection ÷ rider deflection . .	—	—	—	1·01699	—	1·01697	—	1·01950

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings .	869 936 896 919	1046 966 1012 985	862 939 894 920	1036 972 1009 988	867 936 896 918	1045 964 1011 984	860 937 893 919	1040 968 1009 986
Centre of swing .	910·700	995·10	910·45	995·55	910·40	993·80	909·20	994·20
Deflection due to rider or mass. .	—	84·525	—	85·125	—	84·000	—	85·450
Mass deflection ÷ rider deflection .	—	1·01390	—	1·01024	—	1·01533	—	1·01454

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)	*i.* (25)
Scale readings .	863 934 894 916	1043 964 1009 983	859 936 892 917	1033 971 1006 985	863 932 892 915	1040 963 1007 982	857 936 889 915	1035 966 1006 983	862 930 891 914
Centre of swing	908·30	992·60	908·00	993·15	906·60	991·00	906·10	991·40	905·25
Deflection due to rider or mass .	—	84·450	—	85·850	—	84·650	—	85·725	
Mass deflection ÷ rider deflection	—	1·01421	—	1·01538	—	1·01344	—	—	

May 4. Morning. Mean of 10 determinations of M/R = A, Attracted masses in lower position } 1·015050.

TABLE III.—(*continued*).

MAY 4, 1890.—*Same day.* 2.40 to 4.54 P.M. Temperature: in observing room, 13°·9–14°·1; in balance room, 11°·7–11°·75; barometer, 740·3–739·7.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings . .	847 933 883 912	1035 957 1003 977	853 930 886 911	1031 961 1002 979	864 925 890 909	1035 960 1003 977	853 931 885 912	1026 965 1001 980
Centre of swing . .	901·40	986·20	902·00	987·05	902·55	987·10	902·00	987·65
Deflection due to rider or mass . .	—	84·500	—	84·775	—	84·825	—	85·300
Mass deflection ÷ rider deflection .	—	—	—	1·00133	—	1·00251	—	1·00783

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings . .	866 924 891 909	1035 959 1004 977	853 932 886 912	1023 968 1000 982	864 925 889 910	1036 960 1004 978	854 931 886 912	1024 968 1000 982
Centre of swing . .	902·70	987·20	902·80	988·35	902·30	987·75	902·50	988·45
Deflection due to rider or mass . .	—	84·450	—	85·800	—	85·350	—	85·925
Mass deflection ÷ rider deflection .	—	1·01303	—	1·01060	—	1·00601	—	1·00940

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings . .	864 925 890 909	1039 958 1005 978	855 931 887 913	1025 969 1001 982	866 925 891 911	1040 958 1005 978	855 932 888 913	1011 977 996 985
Centre of swing . .	902·55	987·80	903·25	989·20	903·50	987·90	904·00	989·10
Deflection due to rider or mass . .	—	84·900	—	85·825	—	84·150	—	85·225
Mass deflection ÷ rider deflection .	—	1·01148	—	1·01538	—	1·01634	—	1·01232

TABLE III.—(*continued*).

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings . .	864	1036	854	1024	865	1037	854	1031
	927	961	933	970	926	962	934	967
	890	1004	887	1001	892	1005	888	1004
	912	979	914	984	912	980	915	983
Centre of swing . .	903·75	988·10	904·00	989·85	904·35	989·25	904·90	990·55
Deflection due to rider or mass . .	—	84·225	—	85·675	—	84·625	—	85·625
Mass deflection ÷ rider deflection .	—	1·01454	—	1·01481	—	1·01211	—	1·01182

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings . .	864	1039	855	1024	864	1041	856	1025
	928	961	934	972	929	961	934	971
	892	1006	888	1002	891	1006	888	1002
	912	980	914	985	913	980	915	984
Centre of swing . .	904·95	989·50	904·80	991·10	905·00	989·65	905·00	990·65
Deflection due to rider or mass . .	—	84·625	—	86·200	—	84·650	—	85·500
Mass deflection ÷ rider deflection .	—	1·01521	—	1·01846	—	1·01418	—	1·00796

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings . .	866	1038	857	1030	865	1043	858	1035
	927	962	934	960	930	962	934	966
	893	1007	889	1004	893	1008	891	1006
	913	981	915	984	915	981	915	984
Centre of swing . .	905·30	990·40	905·50	991·30	906·60	991·15	906·55	991·55
Deflection due to rider or mass . .	—	85·000	—	85·250	—	84·575	—	84·95
Mass deflection ÷ rider deflection .	—	1·00441	—	1·00546	—	1·00621	—	1·00741

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)	*m.* (56)
Scale readings . .	869	1041	858	1037	867	1041	856	1029
	927	961	935	965	929	963	936	972
	895	1008	891	1007	894	1007	890	1004
	914	982	916	984	914	982	916	986
Centre of swing . .	906·65	990·90	907·00	991·80	906·60	991·05	906·70	992·50
Deflection due to rider or mass . .	—	84·075	—	85·000	—	84·440	—	85·750
Mass deflection ÷ rider deflection .	—	1·01070	—	1·00905	—	1·01155	—	1·01509

TABLE III.—(*continued*).

	i. (57)	*r.* (58)	*i.* (59)	*m.* (60)	*i.* (61)	*r.* (62)	*i.* (63)	*m.* (64)
Scale readings	871 928 895 913	1041 963 1008 982	859 936 890 917	1035 969 1008 985	869 931 895 916	1044 963 1010 983	860 937 892 917	1030 973 1005 986
Centre of swing	906·80	991·50	907·10	993·50	908·20	992·85	908·25	993·30
Deflection due to rider or mass	—	84·550	—	85·850	—	84·625	—	84·775
Mass deflection ÷ rider deflection	—	1·01478	—	1·01493	—	1·00812	—	1·00162

	i. (65)	*r.* (66)	*i.* (67)	*m.* (68)	*i.* (69)	*r.* (70)	*i.* (71)	*m.* (72)
Scale readings	863 935 894 917	1042 965 1010 984	863 935 894 917	1039 969 1008 985	840 949 886 923	1042 965 1009 985	861 937 893 918	1037 971 1009 987
Centre of swing	908·80	993·45	908·80	993·75	909·15	993·25	909·05	995·05
Deflection due to rider or mass	—	84·650	—	84·775	—	84·150	—	85·750
Mass deflection ÷ rider deflection	—	1·00148	—	1·00444	—	1·01322	—	1·01449

	i. (73)	*r.* (74)	*i.* (75)	*m.* (76)	*i.* (77)	*r.* (78)	*i.* (79)	*m.* (80)
Scale readings	865 935 895 918	1045 965 1011 985	860 938 893 919	1038 969 1010 987	868 934 895 918	1045 965 1012 985	863 937 894 919	1041 969 1011 988
Centre of swing	909·55	994·30	909·25	995·00	909·50	994·75	909·80	995·80
Deflection due to rider or mass	—	84·900	—	85·625	—	85·100	—	85·625
Mass deflection ÷ rider deflection	—	1·00928	—	1·00735	—	1·00617	—	1·00765

	i. (81)	*r.* (82)	*i.* (83)	*m.* (84)	*i.* (85)
Scale readings	864 938 895 919	1044 967 1012 986	860 940 894 920	1036 974 1010 989	867 936 896 919
Centre of swing	910·55	995·45	910·65	996·80	910·60
Deflection due to rider or mass	—	84·850	—	86·175	
Mass deflection ÷ rider deflection	—	1·01238	—	—	

TABLE III.—(*continued*).

May 4, afternoon.	Mean of 40 determinations of M/R = A Attracted masses in lower position	1·0100278.
April 30 and May 4.	Mean of 100 determinations of M/R = A Attracted masses in lower position	1·0109685.

III.—ATTRACTED Masses in Upper Position. May 25, 1890; 11.20 to 12.53 noon. Temperature: in observing room, 15°·4–16°; in balance room, 13°·3; barometer, 748·5–748·1 millims. Weather, E. wind, warm, very bright. Time of swing not recorded.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings . .	1071 986 1033 1005	1175 1085 1134 1108	960 1047 998 1025	1049 1028 1041 1034	1003 1021 1010 1017	1173 1085 1134 1107	956 1047 996 1024	1049 1028 1040 1034
Centre of swing . .	1015·90	1116·90	1015·50	1036·20	1014·25	1116·55	1014·25	1035·80
Deflection due to rider or mass .	—	101·200	—	21·325	—	102·300	—	21·500
Mass deflection ÷ rider deflection .	—	—	—	·209582	—	·209311	—	·210320

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings . .	1001 1021 1011 1016	1173 1085 1134 1106	958 1046 996 1024	1049 1028 1038 1033	1002 1020 1010 1015	1173 1084 1134 1105	957 1046 995 1023	1049 1028 1040 1032
Centre of swing . .	1014·35	1116·35	1014·05	1034·70	1013·50	1115·80	1013·25	1035·90
Deflection due to rider or mass .	—	102·150	—	20·925	—	102·425	—	22·850
Mass deflection ÷ rider deflection .	—	·207660	—	·204571	—	·213688	—	·223297

TABLE III.—(*continued*).

	i.	*r.*	*i.*	*m.*	*i.*	*r.*	*i.*	*m.*
	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
Scale readings. . .	1003	1173	956	1048	1001	1172	958	1048
	1019	1082	1043	1025	1019	1081	1042	1026
	1009	1133	994	1038	1008	1131	994	1038
	1015	1104	1023	1032	1014	1103	1021	1030
Centre of swing . .	1012·85	1114·60	1011·90	1033·60	1012·05	1113·10	1011·35	1033·50
Deflection due to rider or mass .	—	102·225	—	21·625	—	101·400	—	22·375
Mass deflection ÷ rider deflection .	—	·217535	—	·212400	—	·216962	—	·220172

	i.	*r.*	*i.*	*m.*	*i.*	*r.*	*i.*	*m.*
	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)
Scale readings. . .	1000	1171	953	1047	1000	1171	955	1046
	1017	1081	1044	1025	1016	1080	1041	1025
	1007	1130	992	1037	1007	1130	992	1036
	1014	1103	1021	1031	1013	1103	1021	1030
Centre of swing . .	1010·90	1112·70	1019·80	1032·90	1010·45	1112·40	1010·00	1032·15
Deflection due to rider or mass .	—	101·850	—	22·275	—	102·175	—	22·050
Mass deflection ÷ rider deflection.	—	·219195	—	·218356	—	·216907	—	·215885

	i.	*r.*	*i.*	*m.*	*i.*	*r.*	*i.*	*m.*
	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)
Scale readings . .	997	1168	952	1046	999	1170	955	1046
	1017	1080	1043	1024	1018	1082	1043	1026
	1006	1131	992	1037	1007	1130	993	1039
	1013	1102	1021	1030	1012	1102	1021	1030
Centre of swing . .	1010·20	1112·40	1010·40	1032·35	1010·70	1112·65	1011·05	1033·75
Deflection due to rider or mass .	—	102·100	—	21·800	—	101·775	—	22·100
Mass deflection ÷ rider deflection.	—	·214740	—	·213857	—	·215672	—	·216858

TABLE III.—(*continued*).

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings. . .	998 1019 1009 1014	1171 1082 1132 1104	955 1043 994 1022	1046 1027 1038 1031	1000 1019 1009 1014	1173 1082 1131 1104	956 1046 995 1023	1048 1028 1039 1032
Centre of swing . .	1012·25	1114·00	1011·65	1033·85	1012·35	1113·80	1013·20	1034·85
Deflection due to rider or mass .	—	102·050	—	21·850	—	101·025	—	21·900
Mass deflection ÷ rider deflection.	—	·215388	—	·215197	—	·216531	—	·216350

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)
Scale readings . .	1001 1019 1009 1015	1172 1083 1132 1105	956 1046 996 1023	1048 1027 1039 1032	1000 1020 1009 1016	1173 1082 1133 1105	956 1044 995 1023
Centre of swing . .	1012·70	1114·60	1013·65	1034·60	1013·15	1114·70	1012·65
Deflection due to rider or mass . .	—	101·425	—	21·200	—	101·800	—
Mass deflection ÷ rider deflection. .	—	·212472	—	·208636	—	—	—

May 25. Morning. Mean of 25 determinations of M/R = *a*
Attracted masses in upper position } ·21446168.

Same day. 3.15 to 4.50 P.M. Temperature: in observing room, 16°·0–16°·25; in balance room, 13°·3–13°·35; barometer, 747·7–747·4 millims.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings. . .	1001 1069 1031 1052	1205 1116 1165 1138	990 1077 1029 1057	1081 1061 1073 1066	1034 1055 1044 1049	1207 1120 1168 1139	991 1080 1030 1059	1083 1062 1076 1068
Centre of swing . .	1044·55	1147·60	1046·30	1068·55	1047·60	1150·50	1048·35	1070·65
Deflection due to rider or mass .	—	102·175	—	21·600	—	102·525	—	21·675
Mass deflection ÷ rider deflection.	—	—	—	·211041	—	·211046	—	·212162

TABLE III.—(*continued*).

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings. . .	1037 1056 1046 1052	1209 1119 1169 1141	995 1082 1030 1059	1086 1066 1078 1071	1039 1058 1048 1054	1213 1121 1172 1145	994 1086 1034 1064	1088 1069 1078 1073
Centre of swing . .	1049·60	1151·10	1049·00	1073·50	1051·60	1154·10	1052·90	1074·90
Deflection due to rider or mass .	—	101·800	—	23·200	—	101·850	—	21·675
Mass deflection ÷ rider deflection.	—	·220408	—	·227842	—	·220299	—	·216738

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings. . .	1045 1059 1050 1056	1212 1126 1175 1147	996 1088 1037 1066	1088 1071 1080 1076	1043 1063 1053 1058	1215 1126 1177 1148	1002 1088 1039 1066	1094 1072 1083 1077
Centre of swing . .	1053·55	1157·20	1055·30	1077·10	1056·30	1158·50	1056·55	1079·25
Deflection due to rider or mass .	—	102·775	—	21·300	—	102·075	—	22·350
Mass deflection ÷ rider deflection.	—	·209073	—	·207957	—	·213813	—	·219575

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings. . .	1044 1064 1053 1061	1217 1127 1178 1150	999 1093 1041 1069	1095 1073 1086 1079	1048 1067 1056 1061	1221 1131 1180 1152	1002 1094 1042 1071	1096 1076 1088 1081
Centre of swing . .	1057·25	1159·80	1059·35	1081·35	1059·70	1162·50	1060·70	1083·55
Deflection due to rider or mass .	—	101·500	—	21·825	—	102·300	—	22·800
Mass deflection ÷ rider deflection.	—	·217611	—	·214181	—	·218112	—	·223147

TABLE III.—(*continued*).

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings. . .	1049 1067 1057 1064	1221 1131 1182 1154	1004 1095 1045 1072	1097 1079 1089 1082	1053 1069 1059 1066	1224 1134 1183 1156	1007 1096 1047 1075	1101 1079 1091 1085
Centre of swing . .	1060·80	1163·75	1062·60	1085·20	1062·95	1165·70	1064·65	1086·90
Deflection due to rider or mass .	—	102·050	—	22·425	—	101·900	—	22·000
Mass deflection ÷ rider deflection.	—	·221583	—	·219907	—	·217983	—	·215898

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings. . .	1054 1072 1061 1068	1226 1135 1185 1157	1009 1096 1048 1076	1101 1081 1093 1086	1054 1073 1064 1068	1225 1135 1187 1159	1008 1099 1048 1080	1102 1081 1094 1089
Centre of swing . .	1065·15	1167·15	1065·35	1088·55	1066·85	1168·40	1067·00	1089·70
Deflection due to rider or mass. .	—	101·90	—	22·450	—	101·475	—	21·625
Mass deflection ÷ rider deflection.	—	·218106	—	·220774	—	·217172	—	·213607

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)
Scale readings . .	1058 1075 1066 1072	1228 1138 1189 1161	1014 1102 1053 1080	1104 1086 1097 1091	1059 1076 1068 1074	1229 1141 1192 1163	1019 1103 1055 1081
Centre of swing . .	1069·15	1170·80	1070·45	1098·00	1070·95	1173·40	1072·15
Deflection due to rider or mass . .	—	101·000	—	22·300	—	101·850	
Mass deflection ÷ rider deflection. .	—	·217458	—	·219867	—	—	

May 25. Afternoon. Mean of 25 determinations of M/R = *a*
Attracted masses in upper position } ·21701412.

Mean of 50 determinations, morning and afternoon, ·2157379.

TABLE III.—(*continued*).

SUMMARY OF SET I.

February 4 . . . . . $a = \cdot 2142212$
May 25 . . . . . $a = \cdot 2157379$
Mean value of . . $a = \cdot 2149791$
April 30 . . . . . $A = 1{\cdot}010905$
May 4 . . . . . $A = 1{\cdot}011032$
Mean value of . . $A = 1{\cdot}0109685$

Therefore $A - a = \cdot 7959894$.

SET II.

All Attracting and Attracted Masses inverted and changed over, each to the other side. The Suspending Rods also reversed and Riders interchanged. The initial position always the higher reading on the scale.

I. ATTRACTED Masses in Lower Position. July 28, 1890; 8.10 to 9.43 P.M. Temperature: in observing room, 17°–16°·9; in balance room, 15°·4; barometer, 747·6–748 millims. Weather fine and calm; wind W.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings. . .	1099 1051 1081 1063	912 1007 951 985	1130 1034 1093 1057	917 1005 952 985	1126 1036 1091 1058	914 1008 951 986	1131 1035 1093 1057	922 1005 954 984
Centre of swing . .	1069·65	971·95	1070·55	972·10	1070·20	972·60	1070·95	973·15
Deflection due to rider or mass .	—	98·150	—	98·275	—	97·975	—	97·575
Mass deflection ÷ rider deflection.	—	—	—	1·00217	—	·99949	—	·99541

Table III.—(*continued*).

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings. . .	1128 1035 1092 1058	913 1010 951 987	1134 1035 1095 1058	924 1006 956 987	1130 1038 1094 1061	915 1013 953 989	1137 1034 1098 1060	919 1012 955 989
Centre of swing . .	1070·50	973·30	1072·25	975·00	1073·05	975·65	1073·80	976·40
Deflection due to rider or mass .	—	98·075	—	97·650	—	97·775	—	97·700
Mass deflection ÷ rider deflection.	—	·99528	—	·99719	—	·99898	—	·99719

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings. . .	1132 1040 1095 1062	917 1014 954 989	1140 1036 1099 1060	924 1009 958 990	1134 1042 1098 1064	916 1016 955 993	1136 1041 1098 1064	960 991 972 983
Centre of swing . .	1074·40	976·55	1075·05	977·40	1076·90	978·20	1076·70	979·05
Deflection due to rider or mass .	—	98·175	—	98·575	—	98·600	—	98·100
Mass deflection ÷ rider deflection.	—	·99962	—	1·00191	—	·99734	—	·99506

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings. . .	1133 1044 1098 1065	916 1018 956 994	1142 1039 1103 1064	925 1013 960 994	1134 1045 1101 1068	918 1019 957 997	1143 1042 1103 1066	925 1018 961 996
Centre of swing . .	1077·60	979·55	1078·65	980·30	1079·80	981·00	1080·00	982·65
Deflection due to rider or mass .	—	98·575	—	98·925	—	98·900	—	97·075
Mass deflection ÷ rider deflection.	—	·99937	—	1·00190	—	·99090	—	·99031

TABLE III.—(*continued*).

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings. . .	1136 1046 1099 1068	924 1019 961 996	1145 1042 1104 1067	930 1016 964 995	1140 1046 1104 1069	918 1022 959 997	1143 1045 1104 1068	928 1018 962 996
Centre of swing . .	1079·45	982·95	1080·75	983·50	1082·00	982·80	1081·80	983·30
Deflection due to rider or mass .	—	97·150	—	97·875	—	99·100	—	98·875
Mass deflection ÷ rider deflection.	—	1·00335	—	·99745	—	·99268	—	1·00051

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings. . .	1138 1047 1104 1071	926 1020 963 997	1146 1045 1107 1069	928 1022 964 998	1142 1047 1106 1071	927 1021 963 999	1144 1047 1107 1070	937 1015 967 996
Centre of swing . .	1082·55	984·45	1083·45	985·75	1083·70	985·10	1084·05	985·10
Deflection due to rider or mass .	—	98·550	—	97·825	—	98·775	—	98·900
Mass deflection ÷ rider deflection.	—	·99797	—	·99151	—	·99582	—	1·00139

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)
Scale readings . .	1140 1049 1105 1072	923 1024 962 999	1148 1045 1108 1071	932 1021 966 998	1141 1050 1106 1072	924 1024 963 1001	1144 1048 1107 1072
Centre of swing . .	1083·95	985·40	1084·35	986·60	1084·80	986·25	1084·75
Deflection due to rider or mass . .	—	98·750	—	97·975	-	98·525	
Mass deflection ÷ rider deflection. .	—	·99684	—	·99328	—	—	

July 28, 1890. Mean of 25 determinations of M/R = A
Attracted masses in lower position } ·9973168.

TABLE III.—(*continued*).

September 17, 1890; 8.0 to 9.31 P.M. Temperature: in observing room, 17°–17°·5; in balance room, 15°·8; barometer, 746·2–746·4 millims. Weather warm, cloudy.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings. . .	1085 1051 1073 1058	908 1004 945 981	1118 1029 1085 1050	921 995 949 978	1109 1036 1081 1053	905 1006 944 981	1126 1026 1087 1050	921 996 951 978
Centre of swing . .	1064·20	967·35	1063·40	966·70	1063·75	967·35	1063·95	967·90
Deflection due to rider or mass .	—	96·450	—	96·875	—	96·500	—	96·450
Mass deflection ÷ rider deflection.	—	—	—	1·00415	—	1·00168	—	·99613

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings. . .	1113 1034 1084 1053	907 1006 944 982	1126 1027 1088 1052	929 993 953 978	1110 1038 1083 1056	910 1007 947 984	1131 1027 1092 1052	934 993 956 979
Centre of swing .	1064·75	967·75	1065·05	968·40	1065·90	969·90	1067·10	970·20
Deflection due to rider or mass .	—	97·150	—	97·075	—	96·600	—	96·850
Mass deflection ÷ rider deflection.	—	·99601	—	1·00206	—	1·00375	—	1·00026

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings. . .	1104 1044 1081 1059	910 1008 947 985	1129 1030 1091 1054	924 1001 953 983	1116 1040 1086 1057	909 1009 947 986	1121 1036 1088 1056	927 1000 956 984
Centre of swing . .	1067·00	970·44	1067·90	971·45	1066·70	970·85	1067·05	972·80
Deflection due to rider or mass .	—	97·050	—	958·50	—	96·025	—	95·550
Mass deflection ÷ rider deflection.	—	·99279	—	·99288	—	·99662	—	·99105

TABLE III.—(*continued*).

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings . . .	1112	914	1131	929	1114	916	1132	934
	1043	1009	1033	1002	1044	1011	1035	1002
	1086	951	1093	957	1087	952	1094	960
	1060	987	1056	984	1061	988	1058	986
Centre of swing . .	1069·65	973·10	1070·15	974·00	1070·70	974·50	1071·70	976·00
Deflection due to rider or mass .	—	96·800	—	96·425	—	96·700	—	96·550
Mass deflection÷rider deflection .	—	·99161	—	·99664	—	·99780	—	·99690

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings . . .	1097	916	1135	942	1119	919	1136	935
	1058	1015	1037	999	1048	1017	1039	1006
	1083	954	1098	965	1093	957	1099	963
	1067	991	1061	986	1066	993	1063	989
Centre of swing . .	1073·40	977·10	1074·80	977·85	1075·80	979·70	1076·25	979·10
Deflection due to rider or mass .	—	97·000	—	97·450	—	96·325	—	97·025
Mass deflection÷rider deflection.	—	1·00000	—	1·00815	—	1·00947	—	1·00362

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings . . .	1122	917	1141	929	1118	921	1141	941
	1048	1018	1038	1011	1053	1019	1041	1009
	1093	956	1101	962	1093	958	1103	966
	1065	994	1062	993	1068	996	1065	993
Centre of swing . .	1076·00	979·45	1076·95	980·65	1078·20	981·40	1079·40	982·65
Deflection due to rider or mass .	—	97·025	—	96·925	—	97·400	—	96·975
Mass deflection÷rider deflection.	—	·99948	—	·99704	—	·99538	-	·99628

TABLE III.—(*continued*).

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)
Scale readings . .	1134 1047 1100 1067	920 1022 958 998	1143 1041 1104 1065	932 1016 964 996	1134 1048 1102 1068	925 1021 962 997	1140 1045 1104 1068
Centre of swing . .	1079·85	982·65	1080·00	983·85	1081·15	984·20	1081·55
Deflection due to rider or mass . .	—	97·275	—	96·725	—	97·150	
Mass deflection ÷ rider deflection. .	—	·99563	—	·99499	—	—	

September 1890. Mean of 25 determinations of M/R = A } ·9984148.
Attracted masses in lower position }

July 28 and September 17 } Mean of 50 determinations of M/R = A, ·9978658.

II. ATTRACTED Masses in Upper Position. September 23, 1890; 7.52 to 9·30 P.M. Temperature: in observing room, 15°·3–15°·4; in balance room, 15°·05. Barometer, 749·8–750·2 millims. Weather, light S.W. wind and clear after heavy showers. Scale readings between about 1100 and 1300; 1000 omitted.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings . .	307 248 285 263	113 210 151 186	329 235 293 257	235 257 243 251	281 261 273 265	112 208 149 185	326 232 290 256	233 256 241 249
Centre of swing . .	271·00	173·10	270·95	248·25	268·35	171·45	268·25	246·60
Deflection due to rider or mass . .	—	97·875	—	21·400	—	96·850	—	21·175
Mass deflection ÷ rider deflection. .	—	—	—	·219797	—	·219799	—	·218581

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings . .	279 260 272 264	110 207 148 183	331 228 290 253	232 255 239 248	277 258 271 262	110 205 147 182	324 229 288 252	230 254 239 247
Centre of swing .	267·30	170·15	266·80	245·15	265·80	168·90	265·55	244·50
Deflection due to rider or mass . .	—	96·900	—	21·150	—	96·775	—	20·225
Mass deflection ÷ rider deflection .	—	·218395	—	·218407	—	·213769	—	·209179

TABLE III.—(*continued*).

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings . .	275 256 269 261	108 204 145 181	323 228 286 251	228 253 237 247	276 255 268 260	107 203 145 179	328 224 287 249	226 252 236 245
Centre of swing .	263·90	167·40	264·10	243·15	263·00	166·65	263·25	241·85
Deflection due to rider or mass . .	—	96·600	—	20·400	—	96·475	—	20·225
Mass deflection ÷ rider deflection .	—	·210274	—	·211317	—	·210547	—	·209652

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings . .	274 254 265 258	106 199 143 176	320 224 283 246	232 245 237 241	271 252 262 255	100 197 138 175	317 221 281 243	222 247 232 241
Centre of swing .	260·90	163·90	260·40	239·85	258·25	160·50	257·90	237·60
Deflection due to rider or mass . .	—	96·750	—	19·475	—	97·575	—	19·100
Mass deflection ÷ rider deflection .	—	·205323	—	·200437	—	·197668	—	·196730

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings . .	266 249 259 254	98 197 136 173	317 219 279 241	221 244 228 237	265 246 257 251	97 193 135 171	311 218 275 240	218 242 226 235
Centre of swing . .	255·50	159·10	255·90	234·20	253·05	157·00	253·30	232·10
Deflection due to rider or mass . .	—	96·600	—	20·275	—	96·175	—	20·150
Mass deflection ÷ rider deflection . .	—	·203804	—	·210351	—	·210164	—	·209271

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings . .	264 243 256 249	95 191 133 168	310 216 273 238	215 239 223 232	261 241 253 246	91 188 128 166	311 210 272 234	212 236 220 229
Centre of swing . .	251·20	154·90	251·40	229·10	248·55	151·10	248·48	226·10
Deflection due to rider or mass . .	—	96·400	—	20·875	—	97·375	—	20·675
Mass deflection ÷ rider deflection . .	—	·212785	—	·215456	—	·213350	—	·213585

TABLE III.—(*continued*).

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)	*m.* (56)
Scale readings . .	257 237 250 243	90 186 127 162	306 208 269 232	211 234 218 227	256 236 248 242	88 184 125 160	303 209 265 231	208 232 216 225
Centre of swing . .	245·15	149·25	245·80	224·10	244·25	147·21	243·95	222·10
Deflection due to rider or mass . .	—	96·225	—	20·925	—	96·900	—	20·350
Mass deflection ÷ rider deflection . .	—	·216160	—	·216699	—	·212977	—	·209956

	i. (57)	*r.* (58)	*i.* (59)	*m.* (60)	*i.* (61)
Scale readings	253 233 246 238	86 180 122 157	301 205 263 227	203 228 213 222	293 204 280* 245
Centre of swing	240·95	144·00	240·95		
Deflection due to rider or mass . . .	—	96·950			

September 23, 1890. Mean of 27 determinations of $M/R = a$ } ·2112753.
Attracted masses in upper position }

September 25, 1890; 7.10 to 8.43 P.M. Temperature: in observing room, 15°–15°·2; in balance room, 15°. Barometer, 760·8, steady. Weather cloudy, with westerly airs. Time of swing 21 seconds. 1000 omitted in scale readings.

	i. (1)	*r.* (2)	*i.* (3)	*m.* (4)	*i.* (5)	*r.* (6)	*i.* (7)	*m.* (8)
Scale readings . .	246 238 243 239	84 179 121 156	301 205 263 228	206 229 215 224	248 233 243 236	82 178 119 156	297 204 260 226	202 228 212 222
Centre of swing . .	240·90	142·90	240·95	220·40	238·95	141·60	238·95	218·10
Deflection due to rider or mass . .	—	98·025	—	19·550	—	97·350	—	20·850
Mass deflection ÷ rider deflection .	—	—	—	·200128	—	·207499	—	·213163

---

* This is a considerable rise, showing either a sudden disturbance or a displacement of the apparatus; possibly the telescope was touched. The rise was maintained and therefore the observations were discontinued.

TABLE III.—(*continued*).

	i. (9)	*r.* (10)	*i.* (11)	*m.* (12)	*i.* (13)	*r.* (14)	*i.* (15)	*m.* (16)
Scale readings . .	248	83	300	204	252	84	303	207
	233	176	203	228	233	182	206	232
	243	119	261	214	245	122	265	217
	236	155	226	224	239	158	228	226
Centre of swing . .	238·95	140·80	239·20	219·50	240·70	144·60	242·45	222·60
Deflection due to rider or mass . .	—	98·275	—	20·450	—	96·975	—	20·825
Mass deflection ÷ rider deflection .	—	·210125	—	·209475	—	·212813	—	·214718

	i. (17)	*r.* (18)	*i.* (19)	*m.* (20)	*i.* (21)	*r.* (22)	*i.* (23)	*m.* (24)
Scale readings . .	255	87	307	210	257	90	271	215
	237	184	207	234	238	184	233	233
	249	125	267	219	251	127	255	222
	242	162	232	228	244	163	241	229
Centre of swing . .	244·40	147·55	244·70	224·65	246·05	148·75	246·70	226·15
Deflection due to rider or mass . .	—	97·000	—	20·725	—	97·625	—	20·725
Mass deflection ÷ rider deflection. .	—	·214175	—	·212974	—	·212292	—	·212564

	i. (25)	*r.* (26)	*i.* (27)	*m.* (28)	*i.* (29)	*r.* (30)	*i.* (31)	*m.* (32)
Scale readings . .	258	90	307	213	262	98	307	215
	241	186	213	237	242	189	215	239
	251	129	270	223	253	131	272	225
	244	164	236	232	246	167	237	233
Centre of swing . .	247·05	150·45	248·60	228·30	248·85	153·00	250·25	230·05
Deflection due to rider or mass . .	—	97·375	—	20·425	—	96·550	—	19·750
Mass deflection ÷ rider deflection. .	—	·211297	—	·210649	—	·208053	—	·204425

	i. (33)	*r.* (34)	*i.* (35)	*m.* (36)	*i.* (37)	*r.* (38)	*i.* (39)	*m.* (40)
Scale readings . .	261	93	312	215	263	96	312	217
	243	189	214	241	245	192	217	243
	253	132	273	225	257	135	275	227
	247	167	237	235	250	168	240	237
Centre of swing .	249·35	153·40	250·80	231·10	252·40	156·05	253·05	233·05
Deflection due to rider or mass . .	—	96·675	—	20·500	—	96·675	—	20·525
Mass deflection ÷ rider deflection .	—	·208172	—	·212051	—	·212180	—	·212254

TABLE III.—(*continued*).

	i. (41)	*r.* (42)	*i.* (43)	*m.* (44)	*i.* (45)	*r.* (46)	*i.* (47)	*m.* (48)
Scale readings . .	264 247 259 251	98 194 136 171	314 219 277 242	220 243 231 238	267 249 260 255	100 197 138 174	321 223 281 246	224 246 233 242
Centre of swing .	254·10	157·85	255·05	235·20	256·25	160·35	259·30	238·05
Deflection due to rider or mass . .	—	96·725	—	20·450	—	97·425	—	21·250
Mass deflection ÷ rider deflection .	—	·211812	—	·210662	—	·214011	—	·218650

	i. (49)	*r.* (50)	*i.* (51)	*m.* (52)	*i.* (53)	*r.* (54)	*i.* (55)
Scale readings	271 252 264 256	102 200 139 176	321 221 282 245	224 247 233 242	271 251 264 256	102 198 140 174	314 226 280 247
Centre of swing	259·30	162·20	259·00	238·40	258·95	161·60	259·50
Deflection due to rider or mass	—	96·950	—	20·575	—	97·625	—
Mass deflection ÷ rider deflection	—	·215704	—	·211487			

September 25, 1890. Mean of 25 determinations of M/R = a } ·21125332.
Attracted masses in upper position }

September 23 and September 25 } Mean of 52 determinations of M/R = a, ·2112647.

SUMMARY OF SET II.

July 28 . . . . . A = ·9973168
September 17 . . . A = ·9984148
Mean value of . . A = ·9978658
September 23 . . . a = ·2112753
September 25 . . . a = ·2112533
Mean value of . . a = ·2112647

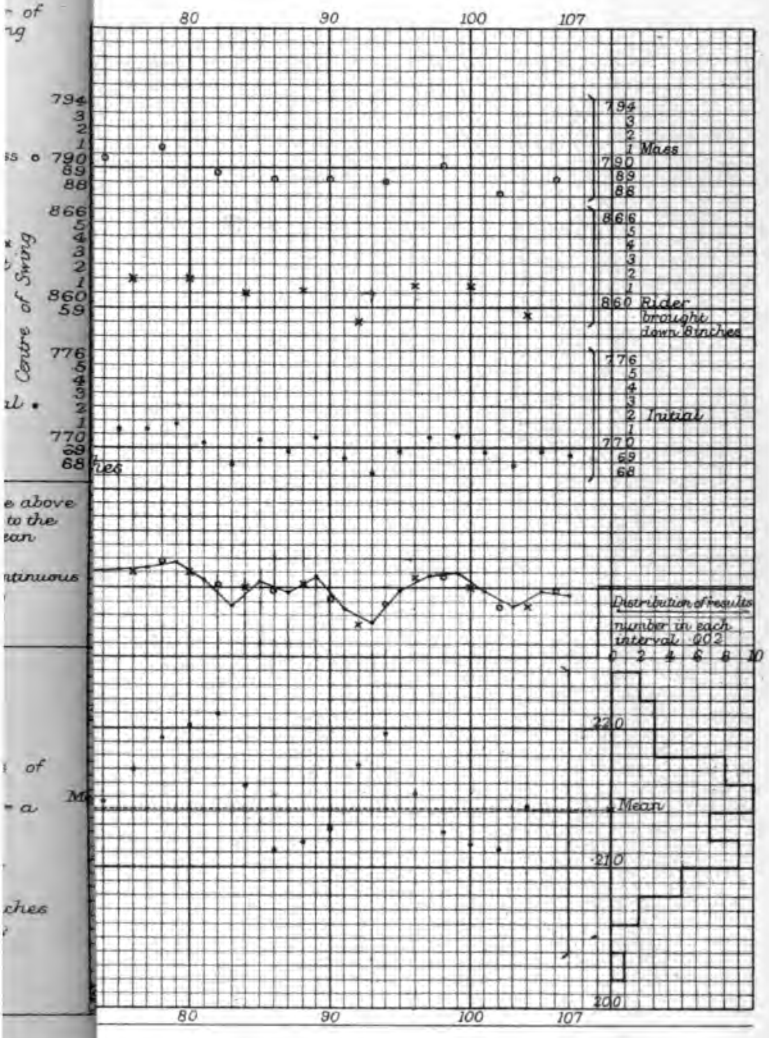
Therefore A − a = ·7866011.

Mean value, giving equal weights to Sets I. and II.,

A − a = ·791295.

PRINTED BY BALLANTYNE, HANSON AND CO.
LONDON AND EDINBURGH

upper position.



West, Newman, lith.

A SELECTION

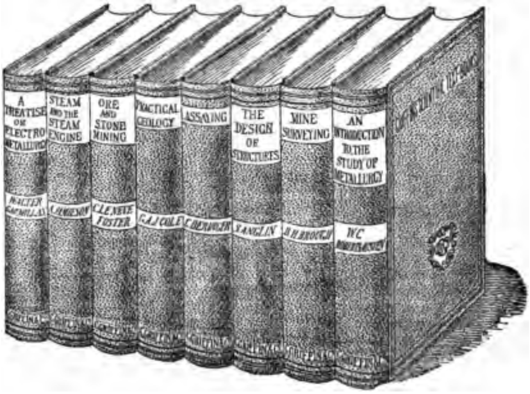
FROM THE

SCIENTIFIC AND TECHNICAL WORKS

PUBLISHED BY

CHARLES GRIFFIN & COMPANY, LIMITED.

(Being Sections 4-10 from Messrs. Griffin's "General Catalogue.")



MESSRS. CHARLES GRIFFIN & COMPANY'S PUBLICATIONS may be obtained through any Bookseller in the United Kingdom, or will be sent Post-free on receipt of a remittance to cover published price. To prevent delay, Orders should be accompanied by a Cheque or Postal Order crossed "SMITH, PAYNE & SMITHS."

*** *For INDEX, see next page.*

☞ MEDICAL and GENERAL CATALOGUES forwarded Post-free on Application.

LONDON:
EXETER STREET, STRAND.

15/2/01

INDEX.

§§ 4-5. Griffin's Standard Publications

FOR

ENGINEERS, ELECTRICIANS, ARCHITECTS, BUILDERS, NAVAL CONSTRUCTORS, AND SURVEYORS.

		PAGE
Applied Mechanics,	RANKINE, BROWNE, JAMIESON, 35, 41,	34
Civil Engineering,	PROF. RANKINE,	35
Bridge-Construction,	PROF. FIDLER,	27
Design of Structures,	S. ANGLIN,	26
Road Making,	T. AITKEN,	77
Sewage Disposal Works,	SANTO CRIMP,	42
Sanitary Engineering,	F. WOOD,	79
Traverse Tables,	R. L. GURDEN,	33
Central Electric Stations,	C. H. WORDINGHAM,	43
Light Railways,	W. H. COLE,	30
Marine Engineering,	A. E. SEATON,	39
Engine-Room Practice,	J. G. LIVERSIDGE,	31
Pocket-Book,	SEATON AND ROUNTHWAITE,	39
Design of Ships,	PROF. HARVARD BILES,	38
Steel Vessels,	T. WALTON,	38
Stability of Ships,	SIR E. J. REED,	38
The Steam-Engine,	RANKINE, JAMIESON,	35, 34
Locomotive Engineering,	W. F. PETTIGREW,	30
Valves and Valve-Gearing	CHAS. HURST,	31
Pumping Machinery,	H. DAVEY,	37
Gas, Oil, and Air-Engines,	BRYAN DONKIN,	28
Gas and Oil Engines,	PROF. WATKINSON,	29
Boilers: Land and Marine,	T. W. TRAILL,	29
,, Steam,	R. D. MUNRO,	32
,, Kitchen,	R. D. MUNRO,	32
,, Heat Efficiency of,	BRYAN DONKIN,	28
Chemistry for Engineers,	BLOUNT & BLOXAM,	41
Fuel and Water,	BROWNE,	41
Machinery and Millwork,	PROF. RANKINE,	35
Hydraulic Machinery,	PROF. ROBINSON,	37
Engineering Drawing,	S. H. WELLS,	33
Lubrication and Lubricants,	ARCHBUTT & DEELEY,	32
Nautical Series,	Ed. by CAPT. BLACKMORE,	45
Rules and Tables,	RANKINE AND JAMIESON,	36
Electrical Pocket-Book,	MUNRO AND JAMIESON,	43
Electrical Price-Book,	H. J. DOWSING,	38
The Calculus for Engineers,	PROF. ROBT. H. SMITH,	40
Measurement Conversion,	PROF. ROBT. H. SMITH,	40
Nystrom's Pocket-Book,	DENNIS MARKS,	32

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

SECOND EDITION, *Revised. With very numerous Diagrams, Examples, and Tables. Large 8vo. Cloth.* 16*s.*

THE DESIGN OF STRUCTURES:

A Practical Treatise on the Building of Bridges, Roofs, &c.

BY S. ANGLIN, C.E.,

Master of Engineering, Royal University of Ireland, late Whitworth Scholar, &c.

The leading features in Mr. Anglin's carefully-planned "Design of Structures" may be briefly summarised as follows:—

1. It supplies the want, long felt among Students of Engineering and Architecture, of a concise Text-book on Structures, requiring on the part of the reader a knowledge of ELEMENTARY MATHEMATICS only.

2. The subject of GRAPHIC STATICS has only of recent years been generally applied in this country to determine the Stresses on Framed Structures; and in too many cases this is done without a knowledge of the principles upon which the science is founded. In Mr. Anglin's work the system is explained from FIRST PRINCIPLES, and the Student will find in it a valuable aid in determining the stresses on all irregularly-framed structures.

3. A large number of PRACTICAL EXAMPLES, such as occur in the every-day experience of the Engineer, are given and carefully worked out, some being solved both analytically and graphically, as a guide to the Student.

4. The chapters devoted to the practical side of the subject, the Strength of Joints, Punching, Drilling, Rivetting, and other processes connected with the manufacture of Bridges, Roofs, and Structural work generally, are the result of MANY YEARS' EXPERIENCE in the bridge-yard; and the information given on this branch of the subject will be found of great value to the practical bridge-builder.

"Students of Engineering will find this Text-Book INVALUABLE."—*Architect.*

"The author has certainly succeeded in producing a THOROUGHLY PRACTICAL Text-Book."—*Builder.*

"We can unhesitatingly recommend this work not only to the Student, as the BEST TEXT-BOOK on the subject, but also to the professional engineer as an EXCEEDINGLY VALUABLE book of reference."—*Mechanical World.*

"This work can be CONFIDENTLY recommended to engineers. The author has wisely chosen to use as little of the higher mathematics as possible, and has thus made his book of REAL USE TO THE PRACTICAL ENGINEER. . . . After careful perusal, we have nothing but praise for the work."—*Nature.*

SECOND EDITION, *Revised.* *Royal 8vo.* *With numerous Illustrations and 13 Lithographic Plates.* *Handsome Cloth.* *Price 30s.*

A PRACTICAL TREATISE ON BRIDGE-CONSTRUCTION:

Being a Text-Book on the Construction of Bridges in Iron and Steel.

FOR THE USE OF STUDENTS, DRAUGHTSMEN, AND ENGINEERS.

BY T. CLAXTON FIDLER, M. INST. C. E.,

Prof. of Engineering, University College, Dundee.

GENERAL CONTENTS.

PART I. — ELEMENTARY STATICS :—Definitions—The Opposition and Balance of Forces—Bending Strain—The Graphic Representation of Bending Moments.

PART II.—GENERAL PRINCIPLES OF BRIDGE-CONSTRUCTION :—The Comparative Anatomy of Bridges—Combined or Composite Bridges—Theoretical Weight of Bridges—On Deflection, or the Curve of a Bended Girder—Continuous Girders.

PART III.—THE STRENGTH OF MATERIALS :—Theoretical Strength of Columns—Design and Construction of Struts—Strength and Construction of Ties—Working Strength of Iron and Steel, and the Working Stress in Bridges—Wöhler's Experiments.

PART IV.—THE DESIGN OF BRIDGES IN DETAIL :—The Load on Bridges—Calculation of Stresses due to the Movable Load—Parallel Girders—Direct Calculation of the Weight of Metal—Parabolic Girders, Polygonal Trusses, and Curved Girders—Suspension Bridges and Arches : Flexible Construction—Rigid Construction—Bowstring Girders used as Arches or as Suspension Bridges—Rigid Arched Ribs or Suspension Ribs—Continuous Girders and Cantilever Bridges—The Niagara Bridge—The Forth Bridge—Wind-Pressure and Wind-Bracing : Modern Experiments.

"Mr. FIDLER'S SUCCESS arises from the combination of EXPERIENCE and SIMPLICITY OF TREATMENT displayed on every page. . . . Theory is kept in subordination to Practice, and his book is, therefore, as useful to girder-makers as to students of Bridge Construction."—*The Architect.*

"Of late years the American treatises on Practical and Applied Mechanics have taken the lead . . . since the opening up of a vast continent has given the American engineer a number of new bridge-problems to solve . . . but we look to the PRESENT TREATISE ON BRIDGE-CONSTRUCTION, and the Forth Bridge, to bring us to the front again."—*Engineer.*

"One of the VERY BEST RECENT WORKS on the Strength of Materials and its application to Bridge-Construction. . . Well repays a careful Study."—*Engineering.*

"An INDISPENSABLE HANDBOOK for the practical Engineer."—*Nature.*

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

Works by BRYAN DONKIN, M.Inst.C.E., M.Inst.Mech.E., &c.

THIRD EDITION, Revised and Enlarged. With additional Illustrations. Large 8vo, Handsome Cloth. 25s.

GAS, OIL, AND AIR ENGINES:

A Practical Text-Book on Internal Combustion Motors without Boiler.

BY BRYAN DONKIN, M.INST.C.E., M.INST.MECH.E.

GENERAL CONTENTS.—**Gas Engines**:—General Description—History and Development—British, French, and German Gas Engines—Gas Production for Motive Power—Theory of the Gas Engine—Chemical Composition of Gas in Gas Engines—Utilisation of Heat—Explosion and Combustion. **Oil Motors**:—History and Development—Various Types—Priestman's and other Oil Engines. **Hot-Air Engines**:—History and Development—Various Types: Stirling's, Ericsson's, &c., &c.

"The BEST BOOK NOW PUBLISHED on Gas, Oil, and Air Engines. . . . Will be of VERY GREAT INTEREST to the numerous practical engineers who have to make themselves familiar with the motor of the day. . . . Mr. Donkin has the advantage of LONG PRACTICAL EXPERIENCE, combined with HIGH SCIENTIFIC AND EXPERIMENTAL KNOWLEDGE, and an accurate perception of the requirements of Engineers."—*The Engineer*.

"We HEARTILY RECOMMEND Mr. Donkin's work. . . . A monument of careful labour. . . . Luminous and comprehensive."—*Journal of Gas Lighting*.

"A thoroughly RELIABLE and EXHAUSTIVE Treatise."—*Engineering*.

In Quarto, Handsome Cloth. With Numerous Plates. 25s.

THE HEAT EFFICIENCY OF STEAM BOILERS

(LAND, MARINE, AND LOCOMOTIVE).

With many Tests and Experiments on different Types of Boilers, as to the Heating Value of Fuels, &c., with Analyses of Gases and Amount of Evaporation, and Suggestions for the Testing of Boilers.

BY BRYAN DONKIN, M.INST.C.E.

GENERAL CONTENTS.—Classification of different Types of Boilers—425 Experiments on English and Foreign Boilers with their Heat Efficiencies shown in Fifty Tables—Fire Grates of Various Types—Mechanical Stokers—Combustion of Fuel in Boilers—Transmission of Heat through Boiler Plates, and their Temperature—Feed Water Heaters, Superheaters, Feed Pumps, &c.—Smoke and its Prevention—Instruments used in Testing Boilers—Marine and Locomotive Boilers—Fuel Testing Stations—Discussion of the Trials and Conclusions—On the Choice of a Boiler, and Testing of Land, Marine, and Locomotive Boilers—Appendices—Bibliography—Index.

With Plates illustrating Progress made during the present Century, and the best Modern Practice.

"A WORK OF REFERENCE AT PRESENT UNIQUE. Will give an answer to almost any question connected with the performance of boilers that it is possible to ask."—*Engineer*.

"Probably the MOST EXHAUSTIVE *resumé* that has ever been collected. A PRACTICAL BOOK by a thoroughly practical man."—*Iron and Coal Trades Review*.

In Crown 8vo, extra, with Numerous Illustrations. [*Shortly.*

GAS AND OIL ENGINES:

AN INTRODUCTORY TEXT-BOOK

On the Theory, Design, Construction, and Testing of Internal Combustion Engines without Boiler.

FOR THE USE OF STUDENTS.

BY

PROF. W. H. WATKINSON, WHIT. SCH., M.INST.MECH.E.,
Glasgow and West of Scotland Technical College.

THIRD EDITION, *Revised and Enlarged. Pocket-Size, Leather,* 12s. 6d.; *also Larger Size for Office Use, Cloth,* 12s. 6d.

Boilers, Marine and Land:

THEIR CONSTRUCTION AND STRENGTH.

A HANDBOOK OF RULES, FORMULÆ, TABLES, &c., RELATIVE TO MATERIAL, SCANTLINGS, AND PRESSURES, SAFETY VALVES, SPRINGS, FITTINGS AND MOUNTINGS, &c.

FOR THE USE OF ENGINEERS, SURVEYORS, BOILER-MAKERS, AND STEAM USERS.

BY T. W. TRAILL, M. INST. C. E., F. E. R. N.,
Late Engineer Surveyor-in-Chief to the Board of Trade.

***** TO THE SECOND AND THIRD EDITIONS MANY NEW TABLES for PRESSURES up to 200 LBS. per SQUARE INCH have been added.**

"**THE MOST VALUABLE WORK** on Boilers published in England."—*Shipping World.*

Contains an ENORMOUS QUANTITY OF INFORMATION arrranged in a very convenient form. . . . A MOST USEFUL VOLUME . . . supplying information to be had nowhere else."—*The Engineer.*

Large 8vo., Handsome Cloth. With numerous Plates reduced from Working Drawings and 280 Illustrations in the Text. 21s.

A MANUAL OF LOCOMOTIVE ENGINEERING:

A Practical Text-Book for the Use of Engine Builders, Designers and Draughtsmen, Railway Engineers, and Students.

BY

WILLIAM FRANK PETTIGREW, M.INST.C.E.

With a Section on American and Continental Engines.

BY ALBERT F. RAVENSHEAR, B.Sc.,

Of Her Majesty's Patent Office.

Contents.—Historical Introduction, 1763-1863.—Modern Locomotives: Simple.—Modern Locomotives: Compound. Primary Consideration in Locomotive Design.—Cylinders, Steam Chests, and Stuffing Boxes.—Pistons, Piston Rods, Crossheads, and Slide Bars.—Connecting and Coupling Rods.—Wheels and Axles, Axle Boxes, Hornblocks, and Bearing Springs.—Balancing.—Valve Gear.—Slide Valves and Valve Gear Details.—Framing, Bogies and Axle Trucks, Radial Axle Boxes.—Boilers.—Smokebox, Blast Pipe, Firebox Fittings.—Boiler Mountings.—Tenders. Railway Brakes.—Lubrication.—Consumption of Fuel, Evaporation and Engine Efficiency.—American Locomotives.—Continental Locomotives.—Repairs, Running, Inspection, and Renewals.—Three Appendices.—Index.

"Likely to remain for many years the STANDARD WORK for those wishing to learn Design."—*Engineer.*

"A most interesting and valuable addition to the bibliography of the Locomotive."—*Railway Official Gazette.*

"We recommend the book as THOROUGHLY PRACTICAL in its character, and MERITING A PLACE IN ANY COLLECTION of . . . works on Locomotive Engineering."—*Railway News.*

"The work CONTAINS ALL THAT CAN BE LEARNT from a book upon such a subject. It will at once rank as THE STANDARD WORK UPON THIS IMPORTANT SUBJECT."—*Railway Magazine.*

In Large 8vo. Handsome Cloth. With Plates and Illustrations. 16s.

LIGHT RAILWAYS

AT HOME AND ABROAD.

BY WILLIAM HENRY COLE, M.INST.C.E.,

Late Deputy-Manager, North-Western Railway, India.

Contents.—Discussion of the Term "Light Railways."—English Railways, Rates, and Farmers.—Light Railways in Belgium, France, Italy, other European Countries, America and the Colonies, India, Ireland.—Road Transport as an alternative.—The Light Railways Act, 1896.—The Question of Gauge.—Construction and Working.—Locomotives and Rolling-Stock.—Light Railways in England, Scotland, and Wales.—Appendices and Index.

"Mr. W. H. Cole has brought together . . . a LARGE AMOUNT of VALUABLE INFORMATION . . . hitherto practically inaccessible to the ordinary reader."—*Times.*

"Will remain, for some time yet a STANDARD WORK in everything relating to Light Railways."—*Engineer.*

"The author has extended practical experience that makes the book lucid and useful. It is EXCEEDINGLY well done."—*Engineering.*

"The whole subject is EXHAUSTIVELY and PRACTICALLY considered. The work can be cordially recommended as INDISPENSABLE to those whose duty it is to become acquainted with one of the prime necessities of the immediate future."—*Railway Official Gazette.*

"THERE COULD BE NO BETTER BOOK of first reference on its subject. All classes of Engineers will welcome its appearance."—*Scotsman.*

Large Crown 8vo. With numerous Illustrations. 6s.

ENGINE-ROOM PRACTICE:

A Handbook for Engineers and Officers in the Royal Navy and Mercantile Marine, Including the Management of the Main and Auxiliary Engines on Board Ship.

By JOHN G. LIVERSIDGE,

Engineer, R.N., A.M.I.C.E., Instructor in Applied Mechanics at the Royal Naval College, Greenwich.

Contents.—General Description of Marine Machinery.—The Conditions of Service and Duties of Engineers of the Royal Navy.—Entry and Conditions of Service of Engineers of the Leading S.S. Companies.—Raising Steam.—Duties of a Steaming Watch on Engines and Boilers.—Shutting off Steam.—Harbour Duties and Watches.—Adjustments and Repairs of Engines.—Preservation and Repairs of "Tank" Boilers.—The Hull and its Fittings.—Cleaning and Painting Machinery.—Reciprocating Pumps, Feed Heaters, and Automatic Feed-Water Regulators.—Evaporators.—Steam Boats.—Electric Light Machinery.—Hydraulic Machinery.—Air-Compressing Pumps.—Refrigerating Machines.—Machinery of Destroyers.—The Management of Water-Tube Boilers.—Regulations for Entry of Assistant Engineers, R.N.—Questions given in Examinations for Promotion of Engineers, R.N.—Regulations respecting Board of Trade Examinations for Engineers, &c.

"The contents CANNOT FAIL TO BE APPRECIATED."—*The Steamship.*

"This VERY USEFUL BOOK. . . . ILLUSTRATIONS are of GREAT IMPORTANCE in a work of this kind, and it is satisfactory to find that SPECIAL ATTENTION has been given in this respect."—*Engineers' Gazette.*

SECOND EDITION, Revised. *With Numerous Illustrations.*
Price 7s. 6d.

VALVES AND VALVE-GEARING:

INCLUDING THE CORLISS VALVE AND TRIP GEARS.

BY

CHARLES HURST, Practical Draughtsman.

"CONCISE explanations illustrated by 115 VERY CLEAR DIAGRAMS and drawings and 4 folding-plates . . . the book fulfils a VALUABLE function."—*Athenæum.*

"MR. HURST'S VALVES and VALVE-GEARING will prove a very valuable aid, and tend to the production of Engines of SCIENTIFIC DESIGN and ECONOMICAL WORKING. . . . Will be largely sought after by Students and Designers."—*Marine Engineer.*

"USEFUL and THOROUGHLY PRACTICAL. Will undoubtedly be found of GREAT VALUE to all concerned with the design of Valve-gearing."—*Mechanical World.*

"Almost EVERY TYPE of VALVE and its gearing is clearly set forth, and illustrated in such a way as to be READILY UNDERSTOOD and PRACTICALLY APPLIED by either the Engineer, Draughtsman, or Student. . . . Should prove both USEFUL and VALUABLE to all Engineers seeking for RELIABLE and CLEAR information on the subject. Its moderate price brings it within the reach of all."—*Industries and Iron.*

"Mr. HURST'S work is ADMIRABLY suited to the needs of the practical mechanic. . . . It is free from any elaborate theoretical discussions, and the explanations of the various types of valve-gear are accompanied by diagrams which render them EASILY UNDERSTOOD."—*The Scientific American.*

Important New Work. 21s.

Large 8vo, Handsome Cloth. With Illustrations, Tables, &c.

Lubrication & Lubricants:

A TREATISE ON THE
THEORY AND PRACTICE OF LUBRICATION
AND ON THE
NATURE, PROPERTIES, AND TESTING OF LUBRICANTS.

By LEONARD ARCHBUTT, F.I.C., F.C.S.,
Chemist to the Midland Railway Company,
AND
R. MOUNTFORD DEELEY, M.I.M.E., F.G.S.,
Midland Railway Locomotive Department.

CONTENTS.—I. Friction of Solids.—II. Liquid Friction or Viscosity, and Plastic Friction.—III. Superficial Tension.—IV. The Theory of Lubrication.—V. Lubricants, their Sources, Preparation, and Properties.—VI. Physical Properties and Methods of Examination of Lubricants.—VII. Chemical Properties and Methods of Examination of Lubricants.—VIII. The Systematic Testing of Lubricants by Physical and Chemical Methods.—IX. The Mechanical Testing of Lubricants.—X. The Design and Lubrication of Bearings.—XI. The Lubrication of Machinery.—INDEX.

" Destined to become a CLASSIC on the subject."—*Industries and Iron.*

"Contains practically ALL THAT IS KNOWN on the subject. Deserves the careful attention of all Engineers."—*Railway Official Guide.*

THIRD EDITION. ***Very fully Illustrated. Cloth, 4s. 6d.***

STEAM-BOILERS:

THEIR DEFECTS, MANAGEMENT, AND CONSTRUCTION.

By R. D. MUNRO,
Chief Engineer of the Scottish Boiler Insurance and Engine Inspection Company.

GENERAL CONTENTS.—I. EXPLOSIONS caused (1) by Overheating of Plates—(2) By Defective and Overloaded Safety Valves—(3) By Corrosion, Internal or External—(4) By Defective Design and Construction (Unsupported Flue Tubes; Unstrengthened Manholes; Defective Staying; Strength of Rivetted Joints; Factor of Safety)—II. CONSTRUCTION OF VERTICAL BOILERS: Shells—Crown Plates and Uptake Tubes—Man-Holes, Mud-Holes, and Fire-Holes—Fireboxes—Mountings—Management—Cleaning—Table of Bursting Pressures of Steel Boilers—Table of Rivetted Joints—Specifications and Drawings of Lancashire Boiler for Working Pressures (*a*) 80 lbs.; (*b*) 200 lbs. per square inch respectively.

" A valuable companion for workmen and engineers engaged about Steam Boilers, ought to be carefully studied, and ALWAYS AT HAND."—*Coll. Guardian.*

" The book is VERY USEFUL, especially to steam users, artisans and young Engineers."—*Engineer.*

BY THE SAME AUTHOR.

KITCHEN BOILER EXPLOSIONS: Why

they Occur, and How to Prevent their Occurrence. A Practical Handbook based on Actual Experiment. With Diagrams and Coloured Plate. Price 3s.

NYSTROM'S POCKET-BOOK OF MECHANICS

AND ENGINEERING. Revised and Corrected by W. DENNIS MARKS, Ph.B., C.E. (YALE S.S.S.), Whitney Professor of Dynamical Engineering, University of Pennsylvania. Pocket Size. Leather, 15s. TWENTY-FIRST EDITION, Revised and greatly enlarged.

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

FIFTH EDITION. Folio, strongly half-bound, 21/.

TRAVERSE TABLES:

Computed to Four Places of Decimals for every Minute of Angle up to 100 of Distance.

For the use of Surveyors and Engineers.

BY

RICHARD LLOYD GURDEN,

Authorised Surveyor for the Governments of New South Wales and Victoria.

⁂ *Published with the Concurrence of the Surveyors-General for New South Wales and Victoria.*

"Those who have experience in exact SURVEY-WORK will best know how to appreciate the enormous amount of labour represented by this valuable book. The computations enable the user to ascertain the sines and cosines for a distance of twelve miles to within half an inch, and this BY REFERENCE TO BUT ONE TABLE, in place of the usual Fifteen minute computations required. This alone is evidence of the assistance which the Tables ensure to every user, and as every Surveyor in active practice has felt the want of such assistance FEW KNOWING OF THEIR PUBLICATION WILL REMAIN WITHOUT THEM."
—*Engineer.*

Engineering Drawing and Design

(A TEXT-BOOK OF).

THIRD EDITION. In Two Parts, Published Separately.

VOL. I.—PRACTICAL GEOMETRY, PLANE, AND SOLID. 3s.

VOL. II.—MACHINE AND ENGINE DRAWING AND DESIGN. 4s. 6d.

BY

SIDNEY H. WELLS, WH.SC.,

A.M.INST.C.E., A.M.INST.MECH.E.,

Principal of the Battersea Polytechnic Institute, and Head of the Engineering Department therein; formerly of the Engineering Departments of the Yorkshire College, Leeds; and Dulwich College, London.

With many Illustrations, specially prepared for the Work, and numerous Examples, for the Use of Students in Technical Schools and Colleges.

"A THOROUGHLY USEFUL WORK, exceedingly well written. For the many Examples and Questions we have nothing but praise."—*Nature.*

"A CAPITAL TEXT-BOOK, arranged on an EXCELLENT SYSTEM, calculated to give an intelligent grasp of the subject, and not the mere faculty of mechanical copying. . . . Mr. Wells shows how to make COMPLETE WORKING-DRAWINGS, discussing fully each step in the design."—*Electrical Review*

"The first book leads EASILY and NATURALLY towards the second, where the technical pupil is brought into contact with large and more complex designs."—*The Schoolmaster.*

WORKS BY
ANDREW JAMIESON, M.INST.C.E., M.I.E.E., F.R.S.E.,

Formerly Professor of Electrical Engineering, The Glasgow and West of Scotland Technical College.

PROFESSOR JAMIESON'S ADVANCED TEXT-BOOKS.

In Large Crown 8vo. Fully Illustrated.

1. STEAM AND STEAM-ENGINES (A Text-Book on). For the Use of Students preparing for Competitive Examinations. With over 200 Illustrations, Folding Plates, and Examination Papers. THIRTEENTH EDITION, Revised. 8/6.

"Professor Jamieson fascinates the reader by his CLEARNESS OF CONCEPTION AND SIMPLICITY OF EXPRESSION. His treatment recalls the lecturing of Faraday."—*Athenæum.*

"The BEST BOOK yet published for the use of Students."—*Engineer.*

"Undoubtedly the MOST VALUABLE AND MOST COMPLETE Hand-book on the subject that now exists."—*Marine Engineer.*

2. MAGNETISM AND ELECTRICITY (An Advanced Text-Book on). Specially arranged for Advanced and "Honours" Students. By Prof. Jamieson, assisted by David Robertson, Jr., B.Sc.

3. APPLIED MECHANICS (An Advanced Text-Book on).

Vol. I.—Comprising Part I.: The Principle of Work and its applications; Part II.: Gearing. Price 7s. 6d. THIRD EDITION.

"FULLY MAINTAINS the reputation of the Author—more we cannot say."—*Pract. Engineer.*

Vol. II.—Comprising Parts III. to VI.: Motion and Energy; Graphic Statics; Strength of Materials; Hydraulics and Hydraulic Machinery. SECOND EDITION. 8s. 6d.

"WELL AND LUCIDLY WRITTEN."—*The Engineer.*

⁂ *Each of the above volumes is complete in itself, and sold separately.*

PROFESSOR JAMIESON'S INTRODUCTORY MANUALS.

Crown 8vo. With Illustrations and Examination Papers.

1. STEAM AND THE STEAM-ENGINE (Elementary Manual of). For First-Year Students. EIGHTH EDITION, Revised. 3/6.

"Quite the RIGHT SORT OF BOOK."—*Engineer.*

"Should be in the hands of EVERY engineering apprentice."—*Practical Engineer.*

2. MAGNETISM AND ELECTRICITY (Elementary Manual of). For First-Year Students. FIFTH EDITION. 3/6.

"A CAPITAL TEXT-BOOK . . . The diagrams are an important feature."—*Schoolmaster.*

"A THOROUGHLY TRUSTWORTHY Text-book. . . . Arrangement as good as well can be. . . . Diagrams are also excellent. . . . The subject throughout treated as an essentially PRACTICAL one, and very clear instructions given."—*Nature.*

3. APPLIED MECHANICS (Elementary Manual of). Specially arranged for First-Year Students. FOURTH EDITION, Revised. 3/6.

"Nothing is taken for granted. . . . The work has VERY HIGH QUALITIES, which may be condensed into the one word 'CLEAR.'"—*Science and Art.*

A POCKET-BOOK of ELECTRICAL RULES and TABLES.

FOR THE USE OF ELECTRICIANS AND ENGINEERS.

Pocket Size. Leather, 8s. 6d. *Fourteenth Edition. See p.* 43.

WORKS BY

W. J. MACQUORN RANKINE, LL.D., F.R.S.,

Late Regius Professor of Civil Engineering in the University of Glasgow.

THOROUGHLY REVISED BY

W. J. MILLAR, C.E.,

Late Secretary to the Institute of Engineers and Shipbuilders in Scotland.

I. A MANUAL OF APPLIED MECHANICS:

Comprising the Principles of Statics and Cinematics, and Theory of Structures, Mechanism, and Machines. With Numerous Diagrams Crown 8vo, cloth, 12s. 6d. FIFTEENTH EDITION.

II. A MANUAL OF CIVIL ENGINEERING:

Comprising Engineering Surveys, Earthwork, Foundations, Masonry, Carpentry, Metal Work, Roads, Railways, Canals, Rivers, Waterworks, Harbours, &c. With Numerous Tables and Illustrations. Crown 8vo cloth, 16s. TWENTY-FIRST EDITION.

III. A MANUAL OF MACHINERY AND MILLWORK:

Comprising the Geometry, Motions, Work, Strength, Construction, and Objects of Machines, &c. Illustrated with nearly 300 Woodcuts. Crown 8vo, cloth, 12s. 6d. SEVENTH EDITION.

IV. A MANUAL OF THE STEAM-ENGINE AND OTHER PRIME MOVERS:

With a Section on GAS, OIL, and AIR ENGINES, by BRYAN DONKIN, M.Inst.C.E. With Folding Plates and Numerous Illustrations. Crown 8vo, cloth, 12s. 6d. FOURTEENTH EDITION.

PROF. RANKINE'S WORKS—(*Continued*).

V. USEFUL RULES AND TABLES:

For Architects, Builders, Engineers, Founders, Mechanics, Shipbuilders, Surveyors, &c. With APPENDIX for the use of ELECTRICAL ENGINEERS. By Professor JAMIESON, F.R.S.E. SEVENTH EDITION. 10s. 6d.

VI. A MECHANICAL TEXT-BOOK:

A Practical and Simple Introduction to the Study of Mechanics. By Professor RANKINE and E. F. BAMBER, C.E. With Numerous Illustrations. Crown 8vo, cloth, 9s. FIFTH EDITION.

⁂ *The* "MECHANICAL TEXT-BOOK" *was designed by* Professor RANKINE *as an* INTRODUCTION *to the above Series of Manuals.*

VII. MISCELLANEOUS SCIENTIFIC PAPERS.

Royal 8vo. Cloth, 31s. 6d.

Part I. Papers relating to Temperature, Elasticity, and Expansion of Vapours, Liquids, and Solids. Part II. Papers on Energy and its Transformations. Part III. Papers on Wave-Forms, Propulsion of Vessels, &c.

With Memoir by Professor TAIT, M.A. Edited by W. J. MILLAR, C.E. With fine Portrait on Steel, Plates, and Diagrams.

"No more enduring Memorial of Professor Rankine could be devised than the publication of these papers in an accessible form. . . . The Collection is most valuable on account of the nature of his discoveries, and the beauty and completeness of his analysis. . . . The Volume exceeds in importance any work in the same department published in our time."—*Architect.*

SHELTON-BEY (W. Vincent, Foreman to the Imperial Ottoman Gun Factories, Constantinople):

THE MECHANIC'S GUIDE: A Hand-Book for Engineers and Artizans. With Copious Tables and Valuable Recipes for Practical Use. Illustrated. *Second Edition.* Crown 8vo. Cloth, 7/6.

IMPORTANT NEW WORK. JUST OUT.

In Large 8*vo, Handsome Cloth. With Frontispiece, several Plates, and over* 250 *Illustrations.* 21*s.*

THE PRINCIPLES AND CONSTRUCTION OF
PUMPING MACHINERY
(STEAM AND WATER PRESSURE).

With Practical Illustrations of ENGINES and PUMPS applied to MINING, TOWN WATER SUPPLY, DRAINAGE of Lands, &c., also Economy and Efficiency Trials of Pumping Machinery.

BY HENRY DAVEY,

Member of the Institution of Civil Engineers, Member of the Institution of Mechanical Engineers, F.G.S., &c.

CONTENTS —Early History of Pumping Engines—Steam Pumping Engines—Pumps and Pump Valves—General Principles of Non-Rotative Pumping Engines—The Cornish Engine, Simple and Compound—Types of Mining Engines—Pit Work—Shaft Sinking—Hydraulic Transmission of Power in Mines—Valve Gears of Pumping Engines—Water Pressure Pumping Engines —Water Works Engines—Pumping Engine Economy and Trials of Pumping Machinery — Centrifugal and other Low-Lift Pumps — Hydraulic Rams, Pumping Mains, &c.— INDEX.

"By the 'one English Engineer who probably knows more about Pumping Machinery than ANY OTHER.' . . . A VOLUME RECORDING THE RESULTS OF LONG EXPERIENCE AND STUDY."—*The Engineer.*

"Undoubtedly THE BEST AND MOST PRACTICAL TREATISE on Pumping Machinery THAT HAS YET BEEN PUBLISHED."—*Mining Journal.*

SECOND EDITION, Revised and Enlarged.
In Large 8vo, Handsome cloth, 34s.

HYDRAULIC POWER
AND
HYDRAULIC MACHINERY.

BY

HENRY ROBINSON, M. INST. C.E., F.G.S.,

FELLOW OF KING'S COLLEGE, LONDON; PROF. OF CIVIL ENGINEERING, KING'S COLLEGE, ETC., ETC.

With numerous Woodcuts, and Sixty-nine Plates.

"A Book of great Professional Usefulness."—*Iron*

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

Royal 8vo, Handsome Cloth, 25s.

THE STABILITY OF SHIPS.

BY

SIR EDWARD J. REED, K.C.B., F.R.S., M.P.,

KNIGHT OF THE IMPERIAL ORDERS OF ST. STANILAUS OF RUSSIA; FRANCIS JOSEPH OF AUSTRIA; MEDJIDIE OF TURKEY; AND RISING SUN OF JAPAN; VICE-PRESIDENT OF THE INSTITUTION OF NAVAL ARCHITECTS.

With numerous Illustrations and Tables.

In order to render the work complete for the purposes of the Shipbuilder, whether at home or abroad, the Methods of Calculation introduced by Mr. F. K. Barnes, Mr. Gray, M. Reech, M. Daymard, and Mr. Benjamin, are all given separately, illustrated by Tables and worked-out examples. The book contains more than 200 Diagrams, and is illustrated by a large number of actual cases, derived from ships of all descriptions, but especially from ships of the Mercantile Marine.

"Sir Edward Reed's 'Stability of Ships' is invaluable. The Naval Architect will find brought together and ready to his hand, a mass of information which he would otherwise have to seek in an almost endless variety of publications, and some of which he would possibly not be able to obtain at all elsewhere."—*Steamship.*

THE DESIGN AND CONSTRUCTION OF SHIPS. By John Harvard Biles, M.Inst.N.A., Professor of Naval Architecture in the University of Glasgow. [*In Preparation.*

At Press. Illustrated with Numerous Plates and Diagrams.

THE CONSTRUCTION AND MAINTENANCE OF VESSELS BUILT OF STEEL.

By THOMAS WALTON, Naval Architect.

Contents.—I. Manufacture of Cast Iron, Wrought Iron, and Steel.—Composition of Iron and Steel, Quality, Strength, Tests, &c. II. Classification of Steel Ships. III. Considerations in making choice of Type of Vessel.—Framing of Ships. IV. Strains experienced by Ships.—Methods of Computing and Comparing Strengths of Ships. V. Construction of Ships.—Alternative Modes of Construction.—Types of Vessels.—Turret, Self Trimming, and Trunk Steamers, &c.—Rivets and Rivetting, Workmanship. VI. Pumping Arrangements. VII. Maintenance.—Prevention of Deterioration in the Hulls of Ships.—Cement, Paint, &c.

Second Edition, Cloth, 8s. 6d. Leather, for the Pocket, 8s. 6d.

GRIFFIN'S ELECTRICAL PRICE-BOOK.

For Electrical, Civil, Marine, and Borough Engineers, Local Authorities, Architects, Railway Contractors, &c., &c.

Edited by H. J. DOWSING,

Member of the Institution of Electrical Engineers; of the Society of Arts; of the London Chamber of Commerce, &c.

"The Electrical Price-Book removes all mystery about the cost of Electrical Power. By its aid the expense that will be entailed by utilising electricity on a large or small scale can be discovered."—*Architect.*

Fourteenth Edition, Revised. Price 21s.

Demy 8vo, Cloth. With Numerous Illustrations, reduced from Working Drawings.

A MANUAL OF
MARINE ENGINEERING:
COMPRISING THE DESIGNING, CONSTRUCTION, AND WORKING OF MARINE MACHINERY.

By A. E. SEATON, M. Inst. C. E., M. Inst. Mech. E., M.Inst.N.A.

GENERAL CONTENTS.

Part I.—Principles of Marine Propulsion.

Part II.—Principles of Steam Engineering.

Part III.—Details of Marine Engines: Design and Calculations for Cylinders, Pistons, Valves, Expansion Valves, &c.

Part IV.—Propellers.

Part V.—Boilers.

Part VI.—Miscellaneous.

⁂ This EDITION includes a Chapter on WATER-TUBE BOILERS, with Illustrations of the leading Types and the Revised Rules of the *Bureau Veritas.*

"In the three-fold capacity of enabling a Student to learn how to design, construct, and work a Marine Steam-Engine, Mr. Seaton's Manual has NO RIVAL."—*Times.*

"The important subject of Marine Engineering is here treated with the THOROUGHNESS that it requires. No department has escaped attention. . . . Gives the results of much close study and practical work."—*Engineering.*

"By far the BEST MANUAL in existence. . . . Gives a complete account of the methods of solving, with the utmost possible economy, the problems before the Marine Engineer."—*Athenæum.*

"The Student, Draughtsman, and Engineer will find this work the MOST VALUABLE HANDBOOK of Reference on the Marine Engine now in existence."—*Marine Engineer.*

SIXTH EDITION, Revised. Pocket-Size, Leather. 8s. 6d.

A POCKET-BOOK OF
MARINE ENGINEERING RULES AND TABLES,
FOR THE USE OF

Marine Engineers, Naval Architects, Designers, Draughtsmen, Superintendents and Others.

BY

A. E. SEATON, M.I.C.E., M.I.Mech.E., M.I.N.A.,

AND

H. M. ROUNTHWAITE, M.I.Mech.E., M.I.N.A.

"ADMIRABLY FULFILS its purpose."—*Marine Engineer.*

WORKS BY PROF. ROBERT H. SMITH, Assoc.M.I.C.E.,

M.I.M.E., M.I.El.E., M.I.Min.E., Whit. Sch., M.Ord.Meiji.

THE CALCULUS FOR ENGINEERS AND PHYSICISTS,

Applied to Technical Problems.

WITH EXTENSIVE

CLASSIFIED REFERENCE LIST OF INTEGRALS.

By PROF. ROBERT H. SMITH.

ASSISTED BY

R. F. MUIRHEAD, M.A., B.Sc.,

Formerly Clark Fellow of Glasgow University, and Lecturer on Mathematics at Mason College.

In Crown 8vo, extra, with Diagrams and Folding-Plate. **8s. 6d.**

"PROF. R. H. SMITH's book will be serviceable in rendering a hard road AS EASY AS PRACTICABLE for the non-mathematical Student and Engineer."—*Athenæum.*

"Interesting diagrams, with practical illustrations of actual occurrence, are to be found here in abundance. THE VERY COMPLETE CLASSIFIED REFERENCE TABLE will prove very useful in saving the time of those who want an integral in a hurry."—*The Engineer.*

MEASUREMENT CONVERSIONS

(English and French):

28 GRAPHIC TABLES OR DIAGRAMS.

Showing at a glance the MUTUAL CONVERSION of MEASUREMENTS in DIFFERENT UNITS

Of Lengths, Areas, Volumes, Weights, Stresses, Densities, Quantities of Work, Horse Powers, Temperatures, &c.

For the use of Engineers, Surveyors, Architects, and Contractors.

In 4to, Boards. 7s. 6d.

⁂ Prof. SMITH'S CONVERSION-TABLES form the most unique and comprehensive collection ever placed before the profession. By their use much time and labour will be saved, and the chances of error in calculation diminished. It is believed that henceforth no Engineer's Office will be considered complete without them.

"The work is INVALUABLE."—*Colliery Guardian.*

"Ought to be in EVERY office where even occasional conversions are required. . . . Prof. SMITH'S TABLES form very EXCELLENT CHECKS on results."—*Electrical Review.*

"Prof. Smith deserves the hearty thanks, not only of the ENGINEER, but of the COMMERCIAL WORLD, for having smoothed the way for the ADOPTION of the METRIC SYSTEM of MEASUREMENT, a subject which is now assuming great importance as a factor in maintaining our HOLD upon FOREIGN TRADE."—*The Machinery Market.*

In Large 8vo. Handsome Cloth. 10s. 6d.

CHEMISTRY FOR ENGINEERS.

BY

BERTRAM BLOUNT, F.I.C., F.C.S., A.I.C.E., Consulting Chemist to the Crown Agents for the Colonies.

AND

A. G. BLOXAM, F.I.C., F.C.S., Consulting Chemist, Head of the Chemistry Department, Goldsmiths' Inst., New Cross.

GENERAL CONTENTS.—Introduction—Chemistry of the Chief Materials of Construction—Sources of Energy—Chemistry of Steam-raising—Chemistry of Lubrication and Lubricants—Metallurgical Processes used in the Winning and Manufacture of Metals.

"The authors have SUCCEEDED beyond all expectation, and have produced a work which should give FRESH POWER to the Engineer and Manufacturer."—*The Times.*

"PRACTICAL THROUGHOUT . . . an ADMIRABLE TEXT-BOOK, useful not only to Students, but to ENGINEERS and MANAGERS OF WORKS in PREVENTING WASTE and IMPROVING PROCESSES."—*Scotsman.*

"A book worthy to take HIGH RANK . . . treatment of the subject of GASEOUS FUEL particularly good. . . . WATER GAS and its production clearly worked out. . . . We WARMLY RECOMMEND the work."—*Journal of Gas Lighting.*

For Companion Volume by the same Authors, see "CHEMISTRY FOR MANUFACTURERS," p. 71.

WORKS BY WALTER R. BROWNE, M.A., M.INST.C.E.,

Late Fellow of Trinity College, Cambridge.

THE STUDENT'S MECHANICS:

An Introduction to the Study of Force and Motion.

With Diagrams. Crown 8vo. Cloth, 4s. 6d.

"Clear in style and practical in method, 'THE STUDENT'S MECHANICS' is cordially to be commended from all points of view."—*Athenæum.*

FOUNDATIONS OF MECHANICS.

Papers reprinted from the *Engineer.* In Crown 8vo, 1s.

Demy 8vo, with Numerous Illustrations, 9s.

FUEL AND WATER:

A Manual for Users of Steam and Water.

BY PROF. FRANZ SCHWACKHÖFER OF VIENNA, AND
WALTER R. BROWNE, M.A., C.E.

GENERAL CONTENTS.—Heat and Combustion—Fuel, Varieties of—Firing Arrangements: Furnace, Flues, Chimney—The Boiler, Choice of—Varieties—Feed-water Heaters—Steam Pipes—Water: Composition, Purification—Prevention of Scale, &c., &c

"The Section on Heat is one of the best and most lucid ever written."—*Engineer.*

"Cannot fail to be valuable to thousands using steam power."—*Railway Engineer*

GRIFFIN'S LOCAL GOVERNMENT HANDBOOKS.

WORKS SUITABLE FOR MUNICIPAL AND COUNTY ENGINEERS, ANALYSTS, AND OTHERS.

Gas Manufacture (The Chemistry of). A Handbook on the Production, Purification, and Testing of Illuminating Gas, and the Assay of the Bye-Product of Gas Manufacture. By W. J. A. Butterfield, M.A., F.I.C., F.C.S. With numerous Illustrations. Handsome Cloth. Second Edition, Revised and Enlarged. 10s. 6d. [See page 77.

"Well deserving a place in every gas engineering library."—*Journal of Gas Lighting.*

Central Electrical Stations: Their Design, Organisation, and Management. By C. H. Wordingham, A.K.C., M.Inst.C.E.

For details see opposite page.

Sewage Disposal Works: A Guide to the Construction of Works for the Prevention of the Pollution by Sewage of Rivers and Estuaries. By W. Santo Crimp, M.Inst.C.E., F.G.S. Second Edition, Revised and Enlarged. Large 8vo, Handsome Cloth. With 37 Plates. Price 30s. [See page 76.

"The most complete and best treatise on the subject which has appeared in our language."—*Edinburgh Medical Journal.*

Calcareous Cements: Their Nature, Preparation, and Uses. With some Remarks upon Cement Testing. By Gilbert Redgrave, Assoc.Inst.C.E. Large Crown 8vo. With Illustrations, Analytical Data, and Appendices on Costs, &c. 8s. 6d. [See page 76.

"Invaluable to student, architect, and engineer."—*Building News.*

Road Making and Maintenance: A Practical Treatise for Engineers, Surveyors, and others. With an Historical Sketch of Ancient and Modern Practice. By Thomas Aitken, Assoc.M.Inst.C.E., Member of the Association of Municipal and County Engineers; Member of the Sanitary Inst.; Surveyor to the County Council of Fife, Cupar Division. Large 8vo, Handsome Cloth, with numerous Plates, Diagrams, and Illustrations. 21s. [See page 77.

"Should be on the reference shelf of every Municipal and County Engineer in the United Kingdom."—*The Surveyor.*

Light Railways at Home and Abroad. By William Henry Cole, M.Inst.C.E., late Deputy Manager, North-Western Railway, India. Large 8vo, Handsome Cloth, Plates and illustrations. 16s. [See page 30.

"Will remain for some time yet a Standard Work in everything relating to Light Railways."—*The Engineer.*

Practical Sanitation: A Handbook for Sanitary Inspectors and others interested in Sanitation. By Geo. Reid, M.D., D.P.H., Fellow of the Sanitary Institute, Medical Officer, Staffordshire County Council. With Appendix on Sanitary Law, by Herbert Manley, M.A., M.B., D.P.H. Seventh Edition. Cloth, 6s. [See page 78.

"A handbook useful to Sanitary Inspectors, and all interested in Sanitary matters."—*Sanitary Record.*

COMPANION VOLUME TO ABOVE.

Sanitary Engineering: A Practical Manual of Town Drainage and Sewage and Refuse Disposal. By Frank Wood, A.M.Inst.C.E., F.G.S. Fully Illustrated. [See page 79.

Dairy Chemistry: A Practical Handbook for Dairy Managers, Chemists, and Analysts. By H. Droop Richmond, F.C.S., Chemist to the Aylesbury Dairy Company. With Tables, Illustrations, &c. Handsome Cloth, 16s. [See page 73.

"The best contribution to the subject that has yet appeared."—*The Lancet.*

Flesh Foods: With Methods for their Chemical, Microscopical, and Bacteriological Examination. A Handbook for Medical Men, Inspectors, Analysts, and others. By C. Ainsworth Mitchell, B.A., F.I.C., Mem. Council Soc. of Public Analysts. With numerous Illustrations and a coloured Plate. 10s. 6d. [See page 78.

"Will save Medical Men, Analysts, Sanitary Inspectors, and others an infinite amount of reference."—*The Lancet.*

Foods: Their Composition and Analysis. By A. Wynter Blyth, M.R.C.S., F.C.S., Public Analyst for the County of Devon. With Tables, Folding Plate, and Frontispiece. Fourth Edition, Revised. 21s. [See page 72.

"An admirable digest of the most recent state of knowledge."—*Chemical News.*

IMPORTANT NEW WORK. READY SHORTLY.

In Large 8vo. Handsome Cloth. Profusely Illustrated with Plates, Diagrams, and Figures.

CENTRAL ELECTRICAL STATIONS:

Their Design, Organisation, and Management.

Including the Generation and Distribution of Electrical Energy.

BY CHAS. H. WORDINGHAM,

A.K.C., M.INST.C.E., M.INST.MECH.E.,

Late Member of the Council of the Institute of Electrical Engineers, and Electrical Engineer to the City of Manchester.

CONTENTS.

Introductory.—Central Station Work as a Profession.—Central Station Supply as an Investment.—The Establishment of a Central Station.—Systems of Supply.—Choice of Site—Architectural Features.—Choice of Plant and General Design.—Boilers.—Systems of Draught and Waste Heat Economy.—Methods of Firing Boilers.—Coal Handling, Weighing, and Storing.—Feeding of Boilers.—The Transmission of Steam.—Generators.—Condensing Appliances.—Switching Gear, Instruments, and Connections.—Distributing Mains.—Drawing-in Systems.—Built-in Systems, Dielectrics.—Insulation, Resistance, and Cost.—Distributing Networks.—Service Mains and Feeders.—Testing of Mains.—Recording and Laying of Mains.—Meters and Appliances on Consumers' Premises.—Standardising and Testing Laboratory.—Secondary Batteries.—Street Lighting.—Cost of Production.—Methods of Charging.—Regulations of Consumer's Installations.—General Organisations of a Central Station.—The Generating Station.—The Mains Department.—The Installation Department.—The Standardising Department.—The Drawing Office, Transforming Stations, and Street Lighting.—The Clerical Department.—The Consumer.—The Routine and Main Laying.—INDEX.

MUNRO & JAMIESON'S ELECTRICAL POCKET-BOOK.

FOURTEENTH EDITION, Revised and Enlarged.

A POCKET-BOOK OF ELECTRICAL RULES & TABLES

FOR THE USE OF ELECTRICIANS AND ENGINEERS.

BY JOHN MUNRO, C.E., & PROF. JAMIESON, M.INST.C.E., F.R.S.E.

With Numerous Diagrams. Pocket Size. Leather, 8s. 6d.

GENERAL CONTENTS.

Units of Measurement.—Measures.—Testing.—Conductors.—Dielectrics.—Submarine Cables.—Telegraphy.—Electro-Chemistry.—Electro-Metallurgy.—Batteries.—Dynamos and Motors.—Transformers.—Electric Lighting.—Miscellaneous.—Logarithms.—Appendices.

"WONDERFULLY PERFECT. . . . Worthy of the highest commendation we can give it."—*Electrician.*

"The STERLING VALUE of Messrs. MUNRO and JAMIESON'S POCKET-BOOK."—*Electrical Review.*

By PROFESSORS J. H. POYNTING & J. J. THOMSON.

In Large 8vo. Fully Illustrated.

A TEXT-BOOK OF PHYSICS:

COMPRISING

PROPERTIES OF MATTER; SOUND; HEAT; MAGNETISM AND ELECTRICITY; AND LIGHT.

BY

J. H. POYNTING,		J. J. THOMSON,
SC.D., F.R.S.,	AND	M.A., F.R.S.,
Late Fellow of Trinity College, Cambridge; Professor of Physics, Mason College, Birmingham.		Fellow of Trinity College, Cambridge; Prof. of Experimental Physics in the University of Cambridge.

VOLUME already published, SECOND EDITION, Price 8s. 6d.

SOUND.

CONTENTS.—The Nature of Sound and its chief Characteristics.—The Velocity of Sound in Air and other Media.—Reflection and Refraction of Sound.—Frequency and Pitch of Notes.—Resonance and Forced Oscillations.—Analysis of Vibrations.—The Transverse Vibrations of Stretched Strings or Wires.—Pipes and other Air Cavities.—Rods.—Plates. —Membranes.—Vibrations maintained by Heat.—Sensitive Flames and Jets.—Musical Sand.—The Superposition of Waves. Index.

"The work . . . may be recommended to anyone desirous of possessing an EASY, UP-TO-DATE STANDARD TREATISE on Acoustics."—*Literature.*

"Very clearly written. . . . The names of the authors are a guarantee of the SCIENTIFIC ACCURACY and UP-TO-DATE CHARACTER of the work."—*Educational Times.*

INTRODUCTORY VOLUME, fully Illustrated. AT PRESS.

PROPERTIES OF MATTER.

CONTENTS.—Gravitation.—The Acceleration of Gravity.—Elasticity.—Stresses and Strains.—Torsion.—Bending of Rods.—Spiral Springs.—Collision.—Compressibility of Liquids.—Pressures and Volumes of Gases.—Thermal Effects Accompanying Strain.—Capillarity.—Surface Tension.—Laplace's Theory of Capillarity.—Diffusion of Liquids.—Diffusion of Gases.—Viscosity of Liquids.

In large 8vo, with Bibliography, Illustrations in the Text, and seven Lithographed Plates. 12s. 6d.

THE MEAN DENSITY OF THE EARTH:

An Essay to which the Adams Prize was adjudged in 1893 in the University of Cambridge.

BY

J. H. POYNTING, Sc.D., F.R.S.,

Late Fellow of Trinity College, Cambridge; Professor of Physics, Mason College, Birmingham.

"An account of this subject cannot fail to be of GREAT and GENERAL INTEREST to the scientific mind. Especially is this the case when the account is given by one who has contributed so considerably as has Prof. Poynting to our present state of knowledge with respect to a very difficult subject. . . . Remarkably has Newton's estimate been verified by Prof. Poynting."—*Athenæum.*

---

§6. GRIFFIN'S NAUTICAL SERIES.

EDITED BY EDW. BLACKMORE,
Master Mariner, First Class Trinity House Certificate, Assoc. Inst. N.A.;
AND WRITTEN, MAINLY, by SAILORS for SAILORS.

"THIS ADMIRABLE SERIES."—*Fairplay*. "A VERY USEFUL SERIES."—*Nature*.

"The volumes of MESSRS. GRIFFIN'S NAUTICAL SERIES may well and profitably be read by ALL interested in our NATIONAL MARITIME PROGRESS."—*Marine Engineer*.

"EVERY SHIP should have the WHOLE SERIES as a REFERENCE LIBRARY. HANDSOMELY BOUND, CLEARLY PRINTED and ILLUSTRATED."—*Liverpool Journ. of Commerce*.

The British Mercantile Marine: An Historical Sketch of its Rise and Development. By the EDITOR, CAPT. BLACKMORE. 3s. 6d.

"Captain Blackmore's SPLENDID BOOK . . . contains paragraphs on every point of interest to the Merchant Marine. The 243 pages of this book are THE MOST VALUABLE to the sea captain that have EVER been COMPILED."—*Merchant Service Review*.

Elementary Seamanship. By D. WILSON-BARKER, Master Mariner, F.R.S.E., F.R.G.S. With numerous Plates, two in Colours, and Frontispiece. SECOND EDITION, Revised. 6s.

'This ADMIRABLE MANUAL, by CAPT. WILSON BARKER, of the 'Worcester,' seems to us PERFECTLY DESIGNED."—*Athenæum*.

Know Your Own Ship: A Simple Explanation of the Stability, Construction, Tonnage, and Freeboard of Ships. By THOS. WALTON, Naval Architect. With numerous Illustrations and additional Chapters on Buoyancy, Trim, and Calculations. FOURTH EDITION, Revised and Enlarged. 7s. 6d.

"MR. WALTON'S book will be found VERY USEFUL."—*The Engineer*.

Navigation: Theoretical and Practical. By D. WILSON-BARKER, Master Mariner, &c., and WILLIAM ALLINGHAM. 3s. 6d.

"PRECISELY the kind of work required for the New Certificates of competency. Candidates will find it INVALUABLE."—*Dundee Advertiser*.

Marine Meteorology: For Officers of the Merchant Navy. By WILLIAM ALLINGHAM, First Class Honours, Navigation, Science and Art Department. With Illustrations, Maps, and Diagrams, and *facsimile* reproduction of log page. 7s. 6d.

"Quite the BEST PUBLICATION on this subject."—*Shipping Gazette*.

Latitude and Longitude: How to find them. By W. J. MILLAR, C.E., late Sec. to the Inst. of Engineers and Shipbuilders in Scotland. 2s.

"Cannot but prove an acquisition to those studying Navigation."—*Marine Engineer*.

Practical Mechanics: Applied to the requirements of the Sailor. By THOS. MACKENZIE, Master Mariner, F.R.A.S. SECOND EDITION, Revised. 3s. 6d.

"WELL WORTH the money . . . EXCEEDINGLY HELPFUL."—*Shipping World*.

Trigonometry: For the Young Sailor, &c. By RICH. C. BUCK, of the Thames Nautical Training College, H.M.S. "Worcester." SECOND EDITION, Revised. Price 3s. 6d.

"This EMINENTLY PRACTICAL and reliable volume."—*Schoolmaster*.

Practical Algebra. By RICH. C. BUCK. Companion Volume to the above, for Sailors and others. Price 3s. 6d.

"It is JUST THE BOOK for the young sailor mindful of progress. —*Nautical Magazine*.

The Legal Duties of Shipmasters. By BENEDICT WM. GINSBURG, M.A., LL.D., of the Inner Temple and Northern Circuit; Barrister-at-Law. Price 4s. 6d.

"INVALUABLE to masters. . . . We can fully recommend it."—*Shipping Gazette*.

A Medical and Surgical Help for Shipmasters. Including First Aid at Sea. By WM. JOHNSON SMITH, F.R.C.S., Principal Medical Officer, Seamen's Hospital, Greenwich. SECOND EDITION, Revised. 6s.

"SOUND, JUDICIOUS, REALLY HELPFUL."—*The Lancet*.

GRIFFIN'S NAUTICAL SERIES.

Price 3s. 6d. Post-free.

THE

British Mercantile Marine.

By EDWARD BLACKMORE,

MASTER MARINER; ASSOCIATE OF THE INSTITUTION OF NAVAL ARCHITECTS;
MEMBER OF THE INSTITUTION OF ENGINEERS AND SHIPBUILDERS
IN SCOTLAND; EDITOR OF GRIFFIN'S "NAUTICAL SERIES."

General Contents.—Historical: From Early Times to 1486—Progress under Henry VIII.—To Death of Mary—During Elizabeth's Reign—Up to the Reign of William III.—The 18th and 19th Centuries—Institution of Examinations — Rise and Progress of Steam Propulsion — Development of Free Trade—Shipping Legislation, 1862 to 1875—"Locksley Hall" Case—Shipmasters' Societies—Loading of Ships—Shipping Legislation, 1884 to 1894—Statistics of Shipping. The Personnel: Shipowners—Officers—Mariners—Duties and Present Position. Education: A Seaman's Education: what it should be—Present Means of Education—Hints. Discipline and Duty—Postscript—The Serious Decrease in the Number of British Seamen, a Matter demanding the Attention of the Nation.

"Interesting and Instructive . . . may be read with profit and enjoyment."—*Glasgow Herald.*

"Every branch of the subject is dealt with in a way which shows that the writer 'knows the ropes' familiarly."—*Scotsman.*

"This admirable book . . . teems with useful information—Should be in the hands of every Sailor."—*Western Morning News.*

Second Edition. *Price 5s. Post-free.*

A MANUAL OF

ELEMENTARY SEAMANSHIP.

BY

D. WILSON-BARKER, Master Mariner; F.R.S.E., F.R.G.S., &c., &c.;
YOUNGER BROTHER OF THE TRINITY HOUSE.

With Frontispiece, Twelve Plates (Two in Colours), and Illustrations in the Text.

General Contents.—The Building of a Ship; Parts of Hull, Masts, &c.—Ropes, Knots, Splicing, &c. — Gear, Lead and Log, &c. — Rigging, Anchors — Sailmaking — The Sails, &c — Handling of Boats under Sail — Signals and Signalling—Rule of the Road—Keeping and Relieving Watch—Points of Etiquette—Glossary of Sea Terms and Phrases—Index.

⁂ The volume contains the new rules of the road.

"This admirable manual, by Capt. Wilson-Barker of the 'Worcester,' seems to us perfectly designed, and holds its place excellently in 'Griffin's Nautical Series.' . . . Although intended for those who are to become Officers of the Merchant Navy, it will be found useful by all yachtsmen."—*Athenæum.*

"Five shillings will be well spent on this little book. Capt. Wilson-Barker knows from experience what a young man wants at the outset of his career."—*The Engineer.*

⁂ For complete List of Griffin's Nautical Series, see p. 45.

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

GRIFFIN'S NAUTICAL SERIES.

Price 3s. 6d. Post-free.

NAVIGATION:

PRACTICAL AND THEORETICAL.

By DAVID WILSON-BARKER, R.N.R., F.R.S.E., &c., &c.,

AND

WILLIAM ALLINGHAM,

FIRST-CLASS HONOURS, NAVIGATION, SCIENCE AND ART DEPARTMENT.

With Numerous Illustrations and Examination Questions.

General Contents.—Definitions—Latitude and Longitude—Instruments of Navigation—Correction of Courses—Plane Sailing—Traverse Sailing—Day's Work — Parallel Sailing — Middle Latitude Sailing — Mercator's Chart—Mercator Sailing—Current Sailing—Position by Bearings—Great Circle Sailing—The Tides—Questions—Appendix: Compass Error—Numerous Useful Hints, &c.—Index.

"Precisely the kind of work required for the New Certificates of competency in grades from Second Mate to extra Master. . . . Candidates will find it invaluable."—*Dundee Advertiser.*

"A capital little book . . . specially adapted to the New Examinations. The Authors are Capt. Wilson-Barker (Captain-Superintendent of the Nautical College, H.M.S. 'Worcester,' who has had great experience in the highest problems of Navigation), and Mr. Allingham, a well-known writer on the Science of Navigation and Nautical Astronomy."—*Shipping World.*

Handsome Cloth. Fully Illustrated. 7s. 6d. Post-free.

MARINE METEOROLOGY,

FOR OFFICERS OF THE MERCHANT NAVY.

By WILLIAM ALLINGHAM,

Joint Author of "Navigation, Theoretical and Practical."

With numerous Plates, Maps, Diagrams, and Illustrations, and a facsimile Reproduction of a Page from an actual Meteorological Log-Book.

SUMMARY OF CONTENTS.

Introductory.—Instruments Used at Sea for Meteorological Purposes.—Meteorological Log-Books.—Atmospheric Pressure.—Air Temperatures.—Sea Temperatures.—Winds.—Wind Force Scales.—History of the Law of Storms.—Hurricanes, Seasons, and Storm Tracks.—Solution of the Cyclone Problem.—Ocean Currents.—Icebergs.—Synchronous Charts.—Dew, Mists, Fogs, and Haze.—Clouds.—Rain, Snow, and Hail.—Mirage, Rainbows, Coronas, Halos, and Meteors.—Lightning, Corposants, and Auroras.—Questions.—Appendix.—Index.

"Quite the best publication, and certainly the most interesting, on this subject ever presented to Nautical men."—*Shipping Gazette.*

⁂ For Complete List of Griffin's Nautical Series, see p. 45.

GRIFFIN'S NAUTICAL SERIES.

SECOND EDITION, REVISED. With Numerous Illustrations. 3s. 6d.

Practical Mechanics:

Applied to the Requirements of the Sailor.

BY THOS. MACKENZIE,

Master Mariner, F.R.A.S.

GENERAL CONTENTS.—Resolution and Composition of Forces—Work done by Machines and Living Agents—The Mechanical Powers: The Lever; Derricks as Bent Levers—The Wheel and Axle: Windlass; Ship's Capstan; Crab Winch—Tackles: the "Old Man"—The Inclined Plane; the Screw—The Centre of Gravity of a Ship and Cargo—Relative Strength of Rope: Steel Wire, Manilla, Hemp, Coir—Derricks and Shears—Calculation of the Cross-breaking Strain of Fir Spar—Centre of Effort of Sails—Hydrostatics: the Diving-bell; Stability of Floating Bodies; the Ship's Pump, &c.

"THIS EXCELLENT BOOK . . . contains a LARGE AMOUNT of **information.**" —*Nature.*

"WELL WORTH the money . . . will be found EXCEEDINGLY HELPFUL."—*Shipping World.*

"NO SHIPS' OFFICERS' BOOKCASE will henceforth be complete without CAPTAIN MACKENZIE'S 'PRACTICAL MECHANICS.' Notwithstanding my many years' experience at sea, it has told me *how much more there is to acquire.*"—(Letter to the Publishers from a Master Mariner).

"I must express my thanks to you for the labour and care you have taken in 'PRACTICAL MECHANICS.' . . . IT IS A LIFE'S EXPERIENCE. . . . What an amount we frequently see wasted by rigging purchases without reason and accidents to spars, &c., &c.! 'PRACTICAL MECHANICS' WOULD SAVE ALL THIS."—(Letter to the Author from another Master Mariner).

WORKS BY RICHARD C. BUCK,

of the Thames Nautical Training College, H.M.S. 'Worcester.'

A Manual of Trigonometry:

With Diagrams, Examples, and Exercises. Post-free 3s. 6d.

SECOND EDITION, Revised and Corrected.

⁂ Mr. Buck's Text-Book has been SPECIALLY PREPARED with a view to the New Examinations of the Board of Trade, in which Trigonometry is an obligatory subject.

"This EMINENTLY PRACTICAL and RELIABLE VOLUME."—*Schoolmaster.*

A Manual of Algebra.

Designed to meet the Requirements of Sailors and others. Price 3s. 6d.

⁂ These elementary works on ALGEBRA and TRIGONOMETRY are written specially for those who will have little opportunity of consulting a Teacher. They are books for "SELF-HELP." All but the simplest explanations have, therefore, been avoided, and ANSWERS to the Exercises are given. Any person may readily, by careful study, become master of their contents, and thus lay the foundation for a further mathematical course, if desired. It is hoped that to the younger Officers of our Mercantile Marine they will be found decidedly serviceable. The Examples and Exercises are taken from the Examination Papers set for the Cadets of the "Worcester."

"Clearly arranged, and well got up. . . . A first-rate Elementary Algebra. —*Nautical Magazine.*

⁂ For complete List of GRIFFIN'S NAUTICAL SERIES, see p. 45.

GRIFFIN'S NAUTICAL SERIES.

In Crown 8vo. Handsome Cloth. 4s. 6d. Post-free.

THE LEGAL DUTIES OF SHIPMASTERS.

BY

BENEDICT WM. GINSBURG, M.A., LL.D. (CANTAB.),

Of the Inner Temple and Northern Circuit; Barrister-at-Law.

General Contents.—The Qualification for the Position of Shipmaster—The Contract with the Shipowner—The Master's Duty in respect of the Crew: Engagement; Apprentices; Discipline; Provisions, Accommodation, and Medical Comforts; Payment of Wages and Discharge—The Master's Duty in respect of the Passengers—The Master's Financial Responsibilities—The Master's Duty in respect of the Cargo—The Master's Duty in Case of Casualty—The Master's Duty to certain Public Authorities—The Master's Duty in relation to Pilots, Signals, Flags, and Light Dues—The Master's Duty upon Arrival at the Port of Discharge—Appendices relative to certain Legal Matters: Board of Trade Certificates, Dietary Scales, Stowage of Grain Cargoes, Load Line Regulations, Life-saving Appliances, Carriage of Cattle at Sea, &c., &c.—Copious Index.

"No intelligent Master should fail to add this to his list of necessary books. A few lines of it may SAVE A LAWYER'S FEE, BESIDES ENDLESS WORRY."—*Liverpool Journal of Commerce.*

"SENSIBLE, plainly written, in CLEAR and NON-TECHNICAL LANGUAGE, and will be found of MUCH SERVICE by the Shipmaster."—*British Trade Review.*

Crown 8vo, with Diagrams. 2s. Post-free.

Latitude and Longitude:

How to Find them.

BY W. J. MILLAR, C.E.,

Late Secretary to the Inst. of Engineers and Shipbuilders in Scotland.

"CONCISELY and CLEARLY WRITTEN . . . cannot but prove an acquisition to those studying Navigation."—*Marine Engineer.*

"Young Seamen will find it HANDY and USEFUL, SIMPLE and CLEAR."—*The Engineer.*

FIRST AID AT SEA.

SECOND EDITION, Revised. With Coloured Plates and Numerous Illustrations, and comprising the latest Regulations Respecting the Carriage of Medical Stores on Board Ship. Post-free. 6s.

A MEDICAL AND SURGICAL HELP

FOR SHIPMASTERS AND OFFICERS
IN THE MERCHANT NAVY.

BY

WM. JOHNSON SMITH, F.R.C.S.,

Principal Medical Officer, Seamen's Hospital, Greenwich.

⁂ The attention of all interested in our Merchant Navy is requested to this exceedingly useful and valuable work. It is needless to say that it is the outcome of many years PRACTICAL EXPERIENCE amongst Seamen.

"SOUND, JUDICIOUS, REALLY HELPFUL."—*The Lancet.*

⁂ For Complete List of GRIFFIN'S NAUTICAL SERIES, see p. 45.

GRIFFIN'S NAUTICAL SERIES.

FOURTH EDITION. *Revised throughout, with additional Chapters on Trim, Buoyancy, and Calculations. Numerous Illustrations.*
Handsome Cloth, Crown 8vo. 7s. 6d.

KNOW YOUR OWN SHIP.

BY THOMAS WALTON, NAVAL ARCHITECT.

SPECIALLY ARRANGED TO SUIT THE REQUIREMENTS OF SHIPS' OFFICERS, SHIPOWNERS, SUPERINTENDENTS, DRAUGHTSMEN, ENGINEERS, AND OTHERS.

This work explains, in a simple manner, such important subjects as:—

- **Displacement,**
- **Deadweight,**
- **Tonnage,**
- **Freeboard,**
- **Moments,**
- **Buoyancy,**
- **Strain,**
- **Structure,**
- **Stability,**
- **Rolling,**
- **Ballasting,**
- **Loading,**
- **Shifting Cargoes,**
- **Admission of Water,**
- **Sail Area,**
- **&c., &c.**

"The little book will be found EXCEEDINGLY HANDY by most officers and officials connected with shipping. . . . Mr. Walton's work will obtain LASTING SUCCESS, because of its unique fitness for those for whom it has been written."—*Shipping World.*

"An EXCELLENT WORK, full of solid instruction and INVALUABLE to every officer of the Mercantile Marine who has his profession at heart."—*Shipping.*

"Not one of the 242 pages could well be spared. It will admirably fulfil its purpose . . . useful to ship owners, ship superintendents, ship draughtsmen, and all interested in shipping."—*Liverpool Journal of Commerce.*

"A mass of VERY USEFUL INFORMATION, accompanied by diagrams and illustrations, is given in a compact form."—*Fairplay.*

"We have found no one statement that we could have wished differently expressed. The matter has, so far as clearness allows, been admirably condensed, and is simple enough to be understood by every seaman."—*Marine Engineer.*

BY THE SAME AUTHOR.

The Construction and Maintenance of Steel Ships.

(See page 38.)

§§ 7-8. Griffin's Geological, Prospecting, Mining, and Metallurgical Publications.

		PAGE
Geology, Stratigraphical,	R. Etheridge, F.R.S., .	52
,, **Physical,** . .	Prof. H. G. Seeley, .	52
,, **Practical Aids,** .	Prof. Grenville Cole,	53
,, **Open Air Studies,** .	,, ,, .	85
Griffin's "New Land" Series,	Ed. by Prof. Cole, .	54
1. **Prospecting for Minerals,**	S. Herbert Cox, A.R.S.M., .	55
2. **Food Supply,** . .	Robt. Bruce, . . .	55
3. **New Lands,** . .	H. R. Mill, D.Sc., F.R.S.E.,	54
4. **Building Construction,**	Prof. Jas. Lyon, . .	54
Ore and Stone Mining, .	Prof. Le Neve Foster, .	56
Elementary Mining, . .	,, ,, .	56
Coal Mining, . .	H. W. Hughes, F.G.S., .	57
Practical Coal Mining, .	G. L. Kerr, M.Inst.M.E., .	58
Petroleum, ...	Redwood and Holloway, .	60
Mine-Surveying, . .	Bennett H. Brough, A.R.S.M.,	56
Blasting and Explosives,	O. Guttmann, A.M.I.C.E., .	57
Mine Accounts, . . .	Prof. J. G. Lawn, . .	58
Mining Engineers' Pkt.-Bk.,	E. R. Field, M.Inst.M.M., .	59
Metallurgy (General Treatise on), . .	Phillips and Bauerman, .	61
,, **(Elementary),**	Prof. Humboldt Sexton, .	66
Assaying, ...	J. J. & C. Beringer, . .	66
Metallurgical Analysis, .	J. J. Morgan, F.C.S., . .	67
Griffin's Metallurgical Series	Ed. by Sir W. Roberts-Austen,	62
1. **Introduction,** . .	Sir W. Roberts-Austen, K.C.B.,	63
2. **Gold, Metallurgy of,**	Dr. Kirke Rose, A.R.S.M.,	63
3. **Lead and Silver,** ,, .	H. F. Collins, A.R.S.M., .	64
4. **Iron, Metallurgy of,** .	Thos. Turner, A.R.S.M., .	65
5. **Steel,** ,, .	F. W. Harbord, A.R.S.M., .	65
6. **Metallurgical Machinery,**	H. C. Jenkins, A.R.S.M., .	65
Getting Gold, . . .	J. C. F. Johnson, F.G.S., .	59
Cyanide Process, . .	James Park, F.G.S., . .	59
Electric Smelting and Refining, . . .	Borchers and McMillan, .	68
Electro-Metallurgy, . .	W. G. McMillan, F.I.C., .	68
Goldsmith and Jeweller's Art,	Thos. B. Wigley, . . .	67

Demy 8vo, Handsome cloth, 18s.

Physical Geology and Palæontology,

ON THE BASIS OF PHILLIPS.

BY

HARRY GOVIER SEELEY, F.R.S.,

PROFESSOR OF GEOGRAPHY IN KING'S COLLEGE, LONDON.

With Frontispiece in Chromo-Lithography, and Illustrations.

"It is impossible to praise too highly the research which PROFESSOR SEELEY'S 'PHYSICAL GEOLOGY' evidences. IT IS FAR MORE THAN A TEXT-BOOK—it is a DIRECTORY to the Student in prosecuting his researches."—*Presidential Address to the Geological Society, 1885, by Rev. Prof. Bonney D.Sc., LL.D., F.R.S.*

"PROFESSOR SEELEY maintains in his 'PHYSICAL GEOLOGY' the high reputation he already deservedly bears as a Teacher."—*Dr. Henry Woodward, F.R.S., in the "Geological Magazine."*

"PROFESSOR SEELEY'S work includes one of the most satisfactory Treatises on Lithology in the English language."—*American Journal of Engineering.*

Demy 8vo, Handsome cloth, 34s.

Stratigraphical Geology & Palæontology,

ON THE BASIS OF PHILLIPS.

BY

ROBERT ETHERIDGE, F.R.S.,

OF THE NATURAL HIST. DEPARTMENT, BRITISH MUSEUM, LATE PALÆONTOLOGIST TO THE GEOLOGICAL SURVEY OF GREAT BRITAIN, PAST PRESIDENT OF THE GEOLOGICAL SOCIETY, ETC.

With Map, Numerous Tables, and Thirty-six Plates.

"No such compendium of geological knowledge has ever been brought together before."—*Westminster Review.*

"If PROF. SEELEY'S volume was remarkable for its originality and the breadth of its views, Mr. ETHERIDGE fully justifies the assertion made in his preface that his book differs in construction and detail from any known manual . . . Must take HIGH RANK AMONG WORKS OF REFERENCE."—*Athenæum.*

OPEN-AIR STUDIES IN GEOLOGY:

An Introduction to Geology Out-of-doors.

BY PROFESSOR GRENVILLE COLE, M.R.I.A., F.G.S.

For details, see Griffin's Introductory Science Series, p. 85.

JUST OUT. *Crown 8vo. Handsome Cloth.* 2*s.* 6*d.*

RESEARCHES ON THE PAST AND PRESENT HISTORY
OF
THE EARTH'S ATMOSPHERE.

Including the latest Discoveries and their Practical Applications.

BY DR. THOMAS LAMB PHIPSON.

PART I.—THE EARTH'S ATMOSPHERE IN REMOTE GEOLOGICAL PERIODS.
PART II.—THE ATMOSPHERE OF OUR PRESENT PERIOD.
APPENDICES ; INDEX.

⁂ Dr. Phipson's work presents, amidst much which is of interest to the Scientist and the General Reader alike, a short *résumé* of his discovery of the origin of Atmospheric Oxygen, the existence of which he attributes wholly to the action of Solar Radiation upon vegetable life. The book will be found replete with much that is new, curious, and interesting, both in connection with Weather Lore, and with Scientific Meteorology.—*Publisher's Note.*

By GRENVILLE A. J. COLE, M.R.I.A., F.G.S.,

Professor of Geology in the Royal College of Science for Ireland.

See also page 85 *and the two following pages* (54, 55).

AIDS IN PRACTICAL GEOLOGY:

WITH A SECTION ON PALÆONTOLOGY.

BY PROFESSOR GRENVILLE COLE, M.R.I.A., F.G.S.

THIRD EDITION, Revised and in part Re-written. With Frontispiece and Illustrations. Cloth, 10s. 6d.

GENERAL CONTENTS.—

PART I.—SAMPLING OF THE EARTH'S CRUST.
PART II.—EXAMINATION OF MINERALS.
PART III.—EXAMINATION OF ROCKS.
PART IV.—EXAMINATION OF FOSSILS.

"Prof. Cole treats of the examination of minerals and rocks in a way that has never been attempted before . . . DESERVING OF THE HIGHEST PRAISE. Here indeed are 'Aids' INNUMERABLE and INVALUABLE. All the directions are given with the utmost clearness and precision."—*Athenæum.*

"That the work deserves its title, that it is full of 'AIDS,' and in the highest degree 'PRACTICAL,' will be the verdict of all who use it."—*Nature.*

"This EXCELLENT MANUAL . . . will be A VERY GREAT HELP. . . . The section on the Examination of Fossils is probably the BEST of its kind yet published. . . . FULL of well-digested information from the newest sources and from personal research."—*Annals of Nat. History.*

GRIFFIN'S "NEW LAND" SERIES.

Practical Hand-Books for the Use of Prospectors, Explorers, Settlers, Colonists, and all Interested in the opening up and Development of New Lands.

Edited by GRENVILLE A. J. COLE, M.R.I.A., F.G.S.,
Professor of Geology in the Royal College of Science for Ireland.

Now Ready. *In Crown 8vo. Handsome Cloth.* **5s.**
With Numerous Maps Specially Drawn and Executed for this Work.

NEW LANDS:

THEIR RESOURCES AND PROSPECTIVE ADVANTAGES.

By HUGH ROBERT MILL, D.Sc., LL.D., F.R.S.E.,
Librarian to the Royal Geographical Society.

Introductory.—The Development of New Lands.—The Dominion of Canada.—Canada, Eastern Provinces.—Canada, Western Providences and Territories.—Newfoundland.—The United States.—Latin America, Mexico.—Latin America, Temperate Brazil and Chili.—Latin America, Argentina.—The Falkland Islands.—Victoria.—New South Wales.—Queensland.—South Australia.—Tasmania.—Western Australia.—New Zealand.—The Resources of South Africa.—Southern Rhodesia.—Index.

"Painstaking . . . complete . . . of great practical assistance."—*The Field.*

"A want admirably supplied. . . . Has the advantage of being written by a professed Geographer."—*Geographical Journal.*

IN PREPARATION.

BUILDING CONSTRUCTION in WOOD, STONE, and CONCRETE. By Jas. Lyon, M.A., Professor of Engineering in the Royal College of Science for Ireland; sometime Superintendent of the Engineering Department in the University of Cambridge; and J. Taylor, A.R.C.S.I.

⁂ Other Volumes, dealing with subjects of **Primary Importance** in the **Examination** and **Utilisation** of **Lands** which have not as yet been fully developed, are in preparation.

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

GRIFFIN'S "NEW LAND" SERIES.

SECOND EDITION, *Revised. With Illustrations. Price in Cloth,* 5s.; *strongly bound in Leather,* 6s. 6d.

PROSPECTING FOR MINERALS.

A Practical Handbook for Prospectors, Explorers, Settlers, and all interested in the Opening up and Development of New Lands.

BY

S. HERBERT COX, Assoc.R.S.M., M.Inst.M.M., F.G.S., &c.

GENERAL CONTENTS.—Introduction and Hints on Geology—The Determination of Minerals: Use of the Blow-pipe, &c.—Rock forming Minerals and Non-Metallic Minerals of Commercial Value: Rock Salt, Borax, Marbles, Lithographic Stone, Quartz and Opal, &c., &c.—Precious Stones and Gems—Stratified Deposits: Coal and Ores—Mineral Veins and Lodes—Irregular Deposits—Dynamics of Lodes: Faults, &c—Alluvial Deposits—Noble Metals: Gold, Platinum, Silver, &c.—Lead—Mercury—Copper—Tin—Zinc—Iron—Nickel, &c.—Sulphur, Antimony, Arsenic, &c.—Combustible Minerals—Petroleum—General Hints on Prospecting—Glossary—Index.

"This ADMIRABLE LITTLE WORK . . . written with SCIENTIFIC ACCURACY in a CLEAR and LUCID style. . . . An IMPORTANT ADDITION to technical literature . . . will be of value not only to the Student, but to the experienced Prospector. . . . If the succeeding volumes of the NEW LAND SERIES are equal in merit to the First, we must congratulate the Publishers on successfully filling up a gap in existing literature.—*Mining Journal.*

"This EXCELLENT HANDBOOK will prove a perfect *Vade-mecum* to those engaged in the practical work of Mining and Metallurgy."—*Times of Africa.*

With many Engravings and Photographs. Handsome Cloth, 4s. 6d.

FOOD SUPPLY.

By ROBERT BRUCE,

Agricultural Superintendent to the Royal Dublin Society.

With Appendix on Preserved Foods by C. A. MITCHELL, B.A., F.I.C.

GENERAL CONTENTS.—Climate and Soil—Drainage and Rotation of Crops—Seeds and Crops—Vegetables and Fruits—Cattle and Cattle-Breeding—Sheep and Sheep Rearing—Pigs—Poultry—Horses—The Dairy—The Farmer's Implements—The Settler's Home.

"BRISTLES WITH INFORMATION."—*Farmers' Gazette.*

"The work is one which will appeal to those intending to become farmers at home or in the Colonies, and who desire to obtain a general idea of the true principles of farming in ALL ITS BRANCHES."—*Journal of the Royal Colonial Inst.*

"A most READABLE and VALUABLE book, and merits an EXTENSIVE SALE."—*Scottish Farmer.*

"Will prove of service in ANY PART OF THE WORLD."—*Nature.*

ORE & STONE MINING.

BY

C. LE NEVE FOSTER, D.Sc., F.R.S.,

PROFESSOR OF MINING, ROYAL COLLEGE OF SCIENCE; H.M. INSPECTOR OF MINES

FOURTH EDITION, Revised, and brought thoroughly up-to-date.
With Frontispiece and 716 Illustrations. **Price 34s.**

"Dr. Foster's book was expected to be EPOCH-MAKING, and it fully justifies such expectation. . . . A MOST ADMIRABLE account of the mode of occurrence of practically ALL KNOWN MINERALS. Probably stands UNRIVALLED for completeness."—*The Mining Journal.*

GENERAL CONTENTS.

INTRODUCTION. Mode of Occurrence of Minerals.—Prospecting.—Boring.—Breaking Ground.—Supporting Excavations.—Exploitation.—Haulage or Transport.—Hoisting or Winding.—Drainage.—Ventilation.—Lighting.—Descent and Ascent.—Dressing—Principles of Employment of Mining Labour.—Legislation affecting Mines and Quarries.—Condition of the Miner.—Accidents.—Index.

"This EPOCH-MAKING work . . . appeals to MEN OF EXPERIENCE no less than to students."—*Berg- und Hüttenmännische Zeitung.*

"This SPLENDID WORK."—*Oesterr. Ztschrft. für Berg- und Hüttenwesen.*

ELEMENTARY MINING AND QUARRYING

(An Introductory Text-book). By Prof. C. LE NEVE FOSTER, F.R.S. [*In Active Preparation.*

A TREATISE ON MINE-SURVEYING:

For the use of Managers of Mines and Collieries, Students at the Royal School of Mines, &c.

BY BENNETT H. BROUGH, F.G.S., ASSOC.R.S.M.,

Formerly Instructor of Mine-Surveying, Royal School of Mines.

EIGHTH EDITION, Revised and Enlarged. With Numerous Diagrams.
Cloth, 7s. 6d.

GENERAL CONTENTS.

General Explanations—Measurement of Distances—Miner's Dial—Variation of the Magnetic-Needle—Surveying with the Magnetic-Needle in presence of Iron—Surveying with the Fixed Needle—German Dial—Theodolite—Traversing Underground—Surface-Surveys with Theodolite—Plotting the Survey—Calculation of Areas—Levelling—Connection of Underground- and Surface-Surveys—Measuring Distances by Telescope—Setting-out—Mine-Surveying Problems—Mine Plans—Applications of Magnetic-Needle in Mining—Photographic Surveying—*Appendices.*

"Has PROVED itself a VALUABLE Text-book; the BEST, if not the only one, in the English language on the subject."—*Mining Journal.*

"No English-speaking Mine Agent or Mining Student will consider his technical library complete without it."—*Nature.*

"A valuable accessory to Surveyors in every department of commercial enterprise. Fully deserves to hold its position as a STANDARD."—*Colliery Guardian.*

A TEXT-BOOK OF COAL-MINING:

FOR THE USE OF COLLIERY MANAGERS AND OTHERS ENGAGED IN COAL-MINING.

BY

HERBERT WILLIAM HUGHES, F.G.S.,

Assoc. Royal School of Mines, General Manager of Sandwell Park Colliery.

FOURTH EDITION, *Revised and Greatly Enlarged. With Numerous Additional Illustrations, mostly reduced from Working Drawings.*

GENERAL CONTENTS.

Geology.—Search for Coal.—Breaking Ground.—Sinking.—Preliminary Operations.—Methods of Working.—Haulage.—Winding.—Pumping.—Ventilation.—Lighting.—Works at Surface.—Preparation of Coal for Market.—INDEX.

"Quite THE BEST BOOK of its kind . . . as PRACTICAL in aim as a book can be . . . The illustrations are EXCELLENT."—*Athenæum.*

"We cordially recommend the work."—*Colliery Guardian.*

"Will soon come to be regarded as the STANDARD WORK of its kind"—*Birmingham Daily Gazette.*

In Large 8vo, with Illustrations and Folding-Plates. 10*s.* 6*d.*

BLASTING:

AND THE USE OF EXPLOSIVES.

A Handbook for Engineers and others Engaged in Mining, Tunnelling, Quarrying, &c.

BY OSCAR GUTTMANN, ASSOC. M. INST. C.E.

Member of the Societies of Civil Engineers and Architects of Vienna and Budapest, Corresponding Member of the Imp. Roy. Geological Institution of Austria, &c.

GENERAL CONTENTS.—Historical Sketch—Blasting Materials—Blasting Powder—Various Powder-mixtures—Gun-cotton—Nitro-glycerine and Dynamite—Other Nitro-compounds—Sprengel's Liquid (acid) Explosives—Other Means of Blasting—Qualities, Dangers, and Handling of Explosives—Choice of Blasting Materials—Apparatus for Measuring Force—Blasting in Fiery Mines—Means of Igniting Charges—Preparation of Blasts—Bore-holes—Machine-drilling—Chamber Mines—Charging of Bore-holes—Determination of the Charge—Blasting in Bore-holes—Firing—Straw and Fuze Firing—Electrical Firing—Substitutes for Electrical Firing—Results of Working—Various Blasting Operations—Quarrying—Blasting Masonry, Iron and Wooden Structures—Blasting in earth, under water, of ice, &c.

"This ADMIRABLE work."—*Colliery Guardian.*

"Should prove a *vade-mecum* to Mining Engineers and all engaged in practical work.—*Iron and Coal Trades Review.*

NEW VOLUME OF GRIFFIN'S MINING SERIES.

Edited by C. LE NEVE FOSTER, D.Sc., F.R.S.,
H.M. Inspector of Mines, Professor of Mining, Royal School of Mines.

In Large 8vo. Price 10s. 6d.

Mine Accounts and Mining Book-keeping.

A Manual for the Use of Students, Managers of Metalliferous Mines and Collieries, Secretaries of Mining Companies. and others interested in Mining.

WITH NUMEROUS EXAMPLES TAKEN FROM THE ACTUAL PRACTICE OF LEADING MINING COMPANIES THROUGHOUT THE WORLD.

BY

JAMES GUNSON LAWN, Assoc.R.S.M., Assoc.Mem.Inst.C.E., F.G.S.,
Professor of Mining at the South African School of Mines, Capetown, Kimberley, and Johannesburg.

GENERAL CONTENTS.—Introduction.—Part I. Engagement and Payment of Workmen.—Part II. Purchases and Sales.—Part III. Working Summaries and Analyses. - Part IV. Ledger, Balance Sheet, and Company Books.—Part V. Reports and Statistics.

"It seems IMPOSSIBLE to suggest how Mr. LAWN'S book could be made more COMPLETE or more VALUABLE, careful, and exhaustive."—*Accountants' Magazine.*

"MR. LAWN'S book should be found of GREAT USE by MINE SECRETARIES and MINE MANAGERS. It consists of five Parts."—*Johannesburg Star.*

JUST OUT. Large Crown 8vo. Handsome Cloth. With over 520 Illustrations in the Text. 12s. 6d.

Practical Coal Mining:

A MANUAL FOR MANAGERS, UNDER-MANAGERS, COLLIERY ENGINEERS, AND OTHERS.

With Worked-out Problems on Haulage, Pumping, Ventilation, &c.

BY GEORGE L. KERR, M.E., M.INST.M.E.,
Colliery Manager.

"The general arrangement of the book . . . closely follows that adopted in the larger works, but a GREAT DEAL OF SPACE IS OCCUPIED ON DESCRIPTION OF DETAILS."—*The Mining Journal.*

"This book JUST MEETS the wants of Students preparing for the Colliery Managers' Examinations. I have decided to use it for our classes here. . . . We have, I believe, the largest mining class in Great Britain."—*The Principal of a Training College.*

With Illustrations and Plates. Handsome Cloth, 6s.

THE CYANIDE PROCESS OF GOLD EXTRACTION.

A Text-Book for the Use of Metallurgists and Students at Schools of Mines, &c.

BY JAMES PARK, F.G.S., M.INST.M.M.,

Late Director Thames School of Mines, and Geological Surveyor and Mining Geologist to the Government of New Zealand.

FIRST ENGLISH EDITION. Thoroughly Revised and Greatly Enlarged from the Third (New Zealand) Edition. With additional details concerning the Siemens-Halske and other recent processes.

CONTENTS.—The MacArthur Process.—Chemistry of the Process.—Laboratory Experiments.—Control Testing and Analysis of Solutions.—Appliances for Cyanide Extraction.—The Actual Extraction by Cyanide.—Application of the Process.—Leaching by Agitation.—Zinc Precipitation of Gold.—The Siemens-Halske Process.—Other Cyanide Processes.—Antidotes for Cyanide Poisoning.—Cyaniding in New Zealand.

"Mr. Park's book deserves to be ranked as amongst the BEST OF EXISTING TREATISES ON THIS SUBJECT."—*Mining Journal.*

SECOND EDITION. *With Illustrations. Cloth,* 3s. 6d.

GETTING GOLD:

A GOLD-MINING HANDBOOK FOR PRACTICAL MEN.

BY J. C. F. JOHNSON, F.G.S., A.I.M.E.,

Life Member Australasian Mine-Managers' Association.

GENERAL CONTENTS.—Introductory: Getting Gold—Gold Prospecting (Alluvial and General)—Lode or Reef Prospecting—The Genesiology of Gold—Auriferous Lodes—Auriferous Drifts—Gold Extraction—Secondary Processes and Lixiviation—Calcination or "Roasting" of Ores—Motor Power and its Transmission—Company Formation and Operations—Rules of Thumb: Mining Appliances and Methods—Selected Data for Mining Men—Australasian Mining Regulations.

"PRACTICAL from beginning to end . . . deals thoroughly with the Prospecting, Sinking, Crushing, and Extraction of gold."—*Brit. Australasian.*

Pocket Size, Strongly bound in Leather. 3s. 6d.

THE MINING ENGINEERS' REPORT BOOK

AND DIRECTORS' AND SHAREHOLDERS' GUIDE TO MINING REPORTS.

BY EDWIN R. FIELD, M.INST.M.M.

With Notes on the Valuation of Mining Property and Tabulating Reports, Useful Tables, &c., and provided with detachable blank pages for MS. Notes.

"An ADMIRABLY compiled book which Mining Engineers and Managers will find EXTREMELY USEFUL."—*Mining Journal.*

PETROLEUM

AND ITS PRODUCTS:

A PRACTICAL TREATISE.

BY

BOVERTON REDWOOD,

F.R.S.E., F.I.C., ASSOC. INST. C.E.,

Hon. Corr. Mem. of the Imperial Russian Technical Society; Mem. of the American Chemical Society; Consulting Adviser to the Corporation of London under the Petroleum Acts, &c., &c.

ASSISTED BY GEO. T. HOLLOWAY, F.I.C., ASSOC. R.C.S.,
And Numerous Contributors.

In Two Volumes, Large 8vo. Price 45s.

With Numerous Maps, Plates, and Illustrations in the Text.

GENERAL CONTENTS.

- **I. General Historical Account of the Petroleum Industry.**
- **II. Geological and Geographical Distribution of Petroleum and Natural Gas.**
- **III. Chemical and Physical Properties of Petroleum.**
- **IV. Origin of Petroleum and Natural Gas.**
- **V. Production of Petroleum, Natural Gas, and Ozokerite.**
- **VI. The Refining of Petroleum.**
- **VII. The Shale Oil and Allied Industries.**
- **VIII. Transport, Storage, and Distribution of Petroleum.**
- **IX. Testing of Petroleum.**
- **X. Application and Uses of Petroleum.**
- **XI. Legislation on Petroleum at Home and Abroad.**
- **XII. Statistics of the Petroleum Production and the Petroleum Trade, obtained from the most trustworthy and official sources.**

"The MOST COMPREHENSIVE AND CONVENIENT ACCOUNT that has yet appeared of a gigantic industry which has made incalculable additions to the comfort of civilised man. . . . The chapter dealing with the arrangement for STORAGE and TRANSPORT of GREAT PRACTICAL INTEREST. . . . The DIGEST of LEGISLATION on the subject cannot but prove of the GREATEST UTILITY."—*The Times.*

"A SPLENDID CONTRIBUTION to our technical literature."—*Chemical News.*

"This THOROUGHLY STANDARD WORK . . . in every way EXCELLENT . . . most fully and ably handled . . . could only have been produced by a man in the very exceptional position of the Author. . . . INDISPENSABLE to all who have to do with Petroleum, its APPLICATIONS, MANUFACTURE, STORAGE, or TRANSPORT."—*Mining Journal.*

"We must concede to Mr. Redwood the distinction of having produced a treatise which must be admitted to the rank of THE INDISPENSABLES. It contains THE LAST WORD that can be said about Petroleum in any of its SCIENTIFIC, TECHNICAL, and LEGAL aspects. It would be difficult to conceive of a more comprehensive and explicit account of the geological conditions associated with the SUPPLY of Petroleum and the very practical question of its AMOUNT and DURATION."—*Journal of Gas Lighting.*

THIRD EDITION. With Folding Plates and Many Illustrations.
Large 8vo. Handsome Cloth. 36s.

ELEMENTS OF METALLURGY:

A PRACTICAL TREATISE ON THE ART OF EXTRACTING METALS FROM THEIR ORES.

BY

J. ARTHUR PHILLIPS, M.INST.C.E., F.C.S., F.G.S., &c.

AND

H. BAUERMAN, V.P.G.S.

GENERAL CONTENTS.

Refractory Materials.
Fire-Clays.
Fuels, &c.
Aluminium.
Copper.
Tin.
Antimony.
Arsenic.
Zinc.
Mercury.
Bismuth.
Lead.
Iron.
Cobalt.
Nickel.
Silver.
Gold.
Platinum.

⁂ Many NOTABLE ADDITIONS, dealing with new Processes and Developments, will be found in the Third Edition.

"Of the THIRD EDITION, we are still able to say that, as a Text-book of Metallurgy, it is THE BEST with which we are acquainted."—*Engineer.*

"The value of this work is almost *inestimable.* There can be no question that the amount of time and labour bestowed on it is enormous. . . . There is certainly no Metallurgical Treatise in the language calculated to prove of such general utility."—*Mining Journal.*

"In this most useful and handsome volume is condensed a large amount of valuable practical knowledge. A careful study of the first division of the book, on Fuels, will be found to be of great value to every one in training for the practical applications of our scientific knowledge to any of our metallurgical operations."—*Athenæum.*

"A work which is equally valuable to the Student as a Text-book, and to the practical Smelter as a Standard Work of Reference. . . . The Illustrations are admirable examples of Wood Engraving."—*Chemical News.*

GRIFFIN'S METALLURGICAL SERIES.

FOURTH EDITION, Revised and Enlarged. Price 15s. Large 8vo,
with numerous Illustrations and Micro-Photographic Plates
of different varieties of Steel.

An Introduction to the Study of

METALLURGY.

BY

Sir W. ROBERTS-AUSTEN, K.C.B., D.C.L., F.R.S.,

Associate of the Royal School of Mines: Chemist and Assayer of the Royal
Mint; Professor of Metallurgy in the Royal College of Science.

GENERAL CONTENTS.—The Relation of Metallurgy to Chemistry.—Physical Properties of Metals.—Alloys.—The Thermal Treatment of Metals.—Fuel and Thermal Measurements.—Materials and Products of Metallurgical Processes.—Furnaces.—Means of Supplying Air to Furnaces.—Thermo-Chemistry.—Typical Metallurgical Processes.—The Micro-Structure of Metals and Alloys.—Economic Considerations.

"No English text-book at all approaches this in the COMPLETENESS with which the most modern views on the subject are dealt with. Professor Austen's volume will be INVALUABLE, not only to the student, but also to those whose knowledge of the art is far advanced."—*Chemical News.*

THIRD EDITION, Revised, Enlarged, and partly Re-written. 21s.
Including the most recent Improvements in the Cyanide Process.
With Frontispiece and numerous Illustrations.

THE METALLURGY OF GOLD.

BY

T. KIRKE ROSE, D.Sc.Lond., Assoc.R.S.M.,

Assistant Assayer of the Royal Mint.

GENERAL CONTENTS.—The Properties of Gold and its Alloys.—Chemistry of Gold.—Mode of Occurrence and Distribution.—Placer Mining.—Shallow Deposits.—Deep Placer Mining.—Quartz Crushing in the Stamp Battery.—Amalgamation.—Other Forms of Crushing and Amalgamating.—Concentration.—Stamp Battery Practice.—Chlorination: The Preparation of Ore.—The Vat Process.—The Barrel Process.—Chlorination Practice in Particular Mills.—The Cyanide Process.—Chemistry of the Process.—Pyritic Smelting.—The Refining and Parting of Gold Bullion—The Assay of Gold Ores.—The Assay of Bullion—Economic Considerations.—Bibliography.

"A COMPREHENSIVE PRACTICAL TREATISE on this important subject."—*The Times.*

"The MOST COMPLETE description of the CHLORINATION PROCESS which has yet been published."—*Mining Journal.*

"Adapted for all who are interested in the Gold Mining Industry, being free from technicalities as far as possible, but is more particularly of value to those engaged in the industry.'—*Cape Times.*

GRIFFIN'S METALLURGICAL SERIES.

EDITED BY SIR W. ROBERTS-AUSTEN, K.C.B., F.R.S., D.C.L.

In Large 8vo. Handsome Cloth. With Illustrations.

IMPORTANT NEW WORK. NOW READY.

In Two Volumes, Each Complete in Itself and Sold Separately.

THE METALLURGY OF LEAD AND SILVER.

BY H. F. COLLINS, ASSOC.R.S.M., M.INST.M.M.

Part I.—LEAD:

A Complete and Exhaustive Treatise on the Manufacture of Lead, with Sections on Smelting and Desilverisation, and Chapters on the Assay and Analysis of the Materials involved. Price 16s.

SUMMARY OF CONTENTS.—Sampling and Assaying Lead and Silver.—Properties and Compounds of Lead.—Lead Ores.—Lead Smelting.—Reverberatories.—Lead Smelting in Hearths.—The Roasting of Lead Ores.—Blast Furnace Smelting; Principles, Practice, and Examples; Products.—Flue Dust, its Composition, Collection and Treatment.—Costs and Losses, Purchase of Ores.—Treatment of Zinc, Lead Sulphides, Desilverisation, Softening and Refining.—The Pattinson Process.—The Parkes Process.—Cupellation and Refining, &c., &c.

"A THOROUGHLY SOUND and useful digest. May with EVERY CONFIDENCE be recommended."—*Mining Journal.*

Part II.—SILVER.

JUST OUT. 16s.

Comprising Details regarding the Sources and Treatment of Silver Ores, together with Descriptions of Plant, Machinery, and Processes of Manufacture, Refining of Bullion, Cost of Working, &c.

SUMMARY OF CONTENTS.—Properties of Silver and its Principal Compounds.—Silver Ores.—The Patio Process.—The Kazo, Fondon, Kröhuke, and Tina Processes.—The Pan Process.—Roast Amalgamation.—Treatment of Tailings and Concentration.—Retorting, Melting, and Assaying.—Chlorodising-Roasting.—The Augustin, Claudet, and Ziervogel Processes.—The Hypo-Sulphide Leaching Process.—Refining.—Matte Smelting.—Pyritic Smelting.—Matte Smelting in Reverberatories.—Silver-Copper Smelting and Refining.—INDEX.

"The author has focussed A LARGE AMOUNT OF VALUABLE INFORMATION into a convenient form. . . . The author has evidently considerable practical experience, and describes the various processes clearly and well."—*Mining Journal.*

GRIFFIN'S METALLURGICAL SERIES.

SECOND EDITION, Revised. Price 16s.

THE METALLURGY OF IRON.

BY THOMAS TURNER, ASSOC.R.S.M., F.I.C.,
Director of Technical Instruction to the Staffordshire County Council.

IN LARGE 8VO, HANDSOME CLOTH, WITH NUMEROUS ILLUSTRATIONS
(MANY FROM PHOTOGRAPHS).

General Contents.—Early History of Iron.—Modern History of Iron.—The Age of Steel.—Chief Iron Ores.—Preparation of Iron Ores.—The Blast Furnace.—The Air used in the Blast Furnace.—Reactions of the Blast Furnace.—The Fuel used in the Blast Furnace.—Slags and Fuxes of Iron Smelting.—Properties of Cast Iron.—Foundry Practice.—Wrought Iron.—Indirect Production of Wrought Iron.—The Puddling Process.—Further Treatment of Wrought Iron.—Corrosion of Iron and Steel.

"A MOST VALUABLE SUMMARY of knowledge relating to every method and stage in the manufacture of cast and wrought iron . . . rich in chemical details. . . . EXHAUSTIVE and THOROUGHLY UP-TO-DATE."—*Bulletin of the American Iron and Steel Association.*

"This is A DELIGHTFUL BOOK, giving, as it does, reliable information on a subject becoming every day more elaborate."—*Colliery Guardian.*

"A THOROUGHLY USEFUL BOOK, which brings the subject UP TO DATE. OF GREAT VALUE to those engaged in the iron industry."—*Mining Journal.*

IN ACTIVE PREPARATION.

New Volume of the "Metallurgical Series."

THE METALLURGY OF STEEL.

BY F. W. HARBORD,
Associate of the Royal School of Mines, Fellow of the Institute of Chemistry, Chemist to the Indian Government, Royal Indian Engineering College, Cooper's Hill.

WITH NUMEROUS DIAGRAMS AND ILLUSTRATIONS OF PLANT AND MACHINERY, REDUCED FROM WORKING DRAWINGS.

IN PREPARATION.

METALLURGICAL MACHINERY:

The Application of Engineering to Metallurgical Problems.

BY HENRY CHARLES JENKINS,
Wh.Sc., Assoc.R.S M., Assoc.M.Inst.C.E.

*** For Details of WORKS ON MINING, see pages 55-59.

A TEXT-BOOK OF ASSAYING:

For the use of Students, Mine Managers, Assayers, &c.

By J. J. BERINGER, F.I.C., F.C.S.,
Public Analyst for, and Lecturer to the Mining Association of, Cornwall.
And C. BERINGER, F.C.S.,
Late Chief Assayer to the Rio Tinto Copper Company, London,

With numerous Tables and Illustrations. Crown 8vo. Cloth, 10/6.
Sixth Edition, Revised and Enlarged.

General Contents. — Part I — Introductory: Manipulation: Sampling; Drying; Calculation of Results—Laboratory-books and Reports. Methods: Dry Gravimetric; Wet Gravimetric—Volumetric Assays: Titrometric, Colorimetric, Gasometric—Weighing and Measuring—Reagents—Formulæ, Equations, &c.—Specific Gravity.

Part II.—Metals: Detection and Assay of Silver, Gold, Platinum, Mercury, Copper, Lead, Thallium, Bismuth, Antimony, Iron, Nickel, Cobalt, Zinc, Cadmium, Tin, Tungsten, Titanium, Manganese, Chromium, &c.—Earths, Alkalies.

Part III.—Non-Metals: Oxygen and Oxides; The Halogens—Sulphur and Sulphates—Arsenic, Phosphorus, Nitrogen—Silicon, Carbon, Boron—Useful Tables.

"A really meritorious work, that may be safely depended upon either for systematic instruction or for reference."—*Nature.*

"This work is one of the best of its kind. . . . Contains all the information that the Assayer will find necessary in the examination of minerals."—*Engineer.*

Second Edition, *Revised. Handsome Cloth. With Numerous Illustrations.* 6s.

A TEXT-BOOK OF
ELEMENTARY METALLURGY.

Including the Author's Practical Laboratory Course.

By A. HUMBOLDT SEXTON, F.I.C., F.C.S.,
Professor of Metallurgy in the Glasgow and West of Scotland Technical College.

GENERAL CONTENTS.—Introduction—Properties of the Metals—Combustion—Fuels—Refractory Materials—Furnaces—Occurrence of the Metals in Nature—Preparation of the Ore for the Smelter—Metallurgical Processes—Iron: Preparation of Pig Iron—Malleable Iron—Steel—Mild Steel—Copper—Lead—Zinc and Tin—Silver—Gold—Mercury—Alloys—Applications of Electricity to Metallurgy—Laboratory Course with Numerous Practical Exercises.

"Just the kind of work for Students commencing the study of Metallurgy, or for Engineering Students requiring a general knowledge of it, or for Engineers in practice who like a handy work of reference. To all three classes we heartily commend the work."—*Practical Engineer.*

"Excellently got-up and well-arranged. . . . Iron and copper well explained by excellent diagrams showing the stages of the process from start to finish. . . . The most novel chapter is that on the many changes wrought in Metallurgical Methods by Electricity."—*Chemical Trade Journal.*

"Possesses the great advantage of giving a Course of Practical Work."—*Mining Journal.*

Large 8vo. Handsome Cloth, 8s. 6d.

The Art of the Goldsmith and Jeweller

A Manual on the Manipulation of Gold in the Various Processes of Goldsmith's Work, and the Manufacture of Personal Ornaments. For Students and Practical Men.

BY THOS. B. WIGLEY,

Headmaster of the Jewellers and Silversmiths' Association Technical School, Birmingham.

ASSISTED BY

J. H. STANSBIE, B.SC. (LOND.), F.I.C.,

Lecturer at the Birmingham Municipal Technical School.

In Large Crown 8vo. With Numerous Illustrations.

General Contents.—Introduction.—The Ancient Goldsmith's Art.—The Metallurgy of Gold.—Prices of Gold, Silver, &c.—Preparation of Alloys.—Melting of Gold.—Rolling and Slitting of Gold.—The Workshop and Tools.—Filigree Wire Drawing.—Manufacture of Personal Ornaments.—Finger Rings.—Mounting and Setting.—Mayoral Chains and Civic Insignia.—Antique Jewellery and its Revival.—Etruscan Work.—Manufacture of Gold Chains.—PRECIOUS STONES.—Cutting Diamonds and other Precious Gems.—Polishing and Finishing.—Chasing, Embossing, and Repoussé Work.—The Colouring and Finishing of Articles of Jewellery.—Enamelling: its History, Processes, and Applicability.—Heraldic Distinctions and Armorial Bearings.—Engraving: its Origin, History, and Processes.—Moulding and Casting of Ornaments, &c.—Fluxes, &c.—Recovery of the Precious Metals from the Waste Products.—Refining Semel and Assaying Semel Bars.—Gilding and Electro Deposition.—Hall-Marking Gold and Silver Plate.—Miscellaneous Useful Information.—Appendix: Technological Examinations.

In Large 8vo. Handsome Cloth. Price 4s.

TABLES FOR QUANTITATIVE METALLURGICAL ANALYSIS.

FOR LABORATORY USE.

BY J. JAMES MORGAN, F.C.S.,

Member Soc. Chem. Industry, Member Cleveland Institute of Engineers.

SUMMARY OF CONTENTS.—Iron Ores.—Steel.—Limestone, &c.—Boiler Incrustations, Clays, and Fire-bricks.—Blast Furnace Slag, &c.—Coal, Coke, and Patent Fuel.—Water.—Gases.—Copper.—Zinc.—Lead.—Alloys.—White Lead.—Atomic Weights.—Factors.—Reagents, &c.

⁂ The above work contains several NOVEL FEATURES, notably the extension, to quantitative analysis, of the PRINCIPLES OF 'GROUP' SEPARATIONS, hitherto chiefly confined to Qualitative work, and will be found to FACILITATE GREATLY the operations of CHEMISTS, ASSAYERS, and others.—*Publisher's Note.*

"The Author may be CONGRATULATED on the way his work has been carried out."—*The Engineer.*

"Will COMMEND ITSELF highly in Laboratory Practice. Its CLEARNESS and PRECISION mark the book out as a highly useful one."—*Mining Journal.*

SECOND EDITION, Revised, Enlarged, and in part Re-written.
With Additional Sections on MODERN THEORIES OF ELECTROLYSIS;
COSTS, &c. Price 10s. 6d.

A TREATISE ON
ELECTRO-METALLURGY:

Embracing the Application of Electrolysis to the Plating, Depositing, Smelting, and Refining of various Metals, and to the Reproduction of Printing Surfaces and Art-Work, &c.

BY

WALTER G. McMILLAN, F.I.C., F.C.S.,

Secretary to the Institution of Electrical Engineers; late Lecturer in Metallurgy at Mason College, Birmingham.

With numerous Illustrations. Large Crown 8vo. Cloth.

"This excellent treatise, . . . one of the BEST and MOST COMPLETE manuals hitherto published on Electro-Metallurgy."—*Electrical Review.*

"This work will be a STANDARD."—*Jeweller.*

"Any metallurgical process which REDUCES the COST of production must of necessity prove of great commercial importance. . . . We recommend this manual to ALL who are interested in the PRACTICAL APPLICATION of electrolytic processes."—*Nature.*

In large 8vo. With Numerous Illustrations and Three Folding-Plates.
Price 21s.

ELECTRIC SMELTING & REFINING:

A Practical Manual of the Extraction and Treatment of Metals by Electrical Methods.

Being the "ELEKTRO-METALLURGIE" of DR. W. BORCHERS.

Translated from the Second Edition by WALTER G. McMILLAN, F.I.C., F.C.S.

CONTENTS.

PART I.—ALKALIES AND ALKALINE EARTH METALS: Magnesium, Lithium, Beryllium, Sodium, Potassium, Calcium, Strontium, Barium, the Carbides of the Alkaline Earth Metals.

PART II.—THE EARTH METALS: Aluminium, Cerium, Lanthanum, Didymium.

PART III.—THE HEAVY METALS: Copper, Silver, Gold, Zinc and Cadmium, Mercury, Tin, Lead, Bismuth, Antimony, Chromium, Molybdenum, Tungsten, Uranium, Manganese, Iron, Nickel, and Cobalt, the Platinum Group.

"COMPREHENSIVE and AUTHORITATIVE . . . not only FULL of VALUABLE INFORMATION, but gives evidence of a THOROUGH INSIGHT into the technical VALUE and POSSIBILITIES of all the methods discussed."—*The Electrician.*

"Dr. BORCHERS' WELL-KNOWN WORK . . . must OF NECESSITY BE ACQUIRED by every one interested in the subject. EXCELLENTLY put into English with additional matter by Mr. McMILLAN."—*Nature.*

"Will be of GREAT SERVICE to the practical man and the Student."—*Electric Smelting*

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

§9. Griffin's Chemical and Technological Publications.

For Metallurgy and Electro-Metallurgy, see previous Section.

		PAGE
Inorganic Chemistry, .	Profs. Dupré and Hake,	70
Quantitative Analysis, .	Prof. Humboldt Sexton,	70
Qualitative ,, .	,, ,,	70
Chemical Recreations, .	J. J. Griffin, . .	21
Chemistry for Engineers,	MM. Blount and Bloxam,	71
,, ,, Manufacturers,	,, ,,	71
Tables for Chemists and Manufacturers, . .	Prof. Castell-Evans, .	73
Agricultural Chemistry, .	Prof. J. M. Munro, .	73
Dairy Chemistry, . .	H. D. Richmond, .	73
Flesh Foods, . . .	C. A. Mitchell, .	78
Foods, Analysis of, . .	A. Wynter Blyth, .	72
Poisons, Detection of, .	,, .	72
Practical Sanitation, .	Dr. G. Reid, . .	78
Sanitary Engineering, .	F. Wood, . . .	79
Technical Mycology (The Fermentation Industries)	Lafar and Salter, .	74
Brewing, ...	Dr. W. J. Sykes, .	75
Sewage Disposal, . .	Santo Crimp, . .	76
Cements, ...	G. R. Redgrave, .	76
Road Making, . . .	Thos. Aitken, . .	77
Gas Manufacture, . .	W. Atkinson Butterfield,	77
Petroleum, . .	MM. Redwood and Holloway,	60
Oils, Soaps, Candles, .	Dr. Alder Wright, .	80
Lubricants and Lubrication, ...	MM. Archbutt and Deeley, . .	32
Painters' Colours, Oils, and Varnishes, . .	G. H. Hurst, . .	80
Painting and Decorating,	W. J. Pearce, . .	81
Photography, . . .	A. Brothers, . .	81
Dyeing, ...	MM. Knecht and Rawson,	82
Textile Printing, . .	Seymour Rothwell, .	83
Textile Fibres, . . .	W. J. Hannan, . .	83
Dyestuffs, ...	Rawson, Gardner, and Laycock, . . .	82
Dyeing and Cleaning, .	G. H. Hurst, . .	83
Bleaching and Calico-Printing, . . .	Geo. Duerr, . .	83

A SHORT MANUAL OF
INORGANIC CHEMISTRY.

BY

A. DUPRÉ, Ph.D., F.R.S.,

AND

WILSON HAKE, Ph.D., F.I.C., F.C.S.,
Of the Westminster Hospital Medical School.

THIRD EDITION, Revised, Enlarged, and brought up-to-date.

"A well-written, clear and accurate Elementary Manual of Inorganic Chemistry. . . . We agree heartily with the system adopted by Drs. Dupré and Hake. WILL MAKE EXPERIMENTAL WORK TREBLY INTERESTING BECAUSE INTELLIGIBLE."—*Saturday Review.*

There is no question that, given the PERFECT GROUNDING of the Student in his Science, the remainder comes afterwards to him in a manner much more simple and easily acquired The work IS AN EXAMPLE OF THE ADVANTAGES OF THE SYSTEMATIC TREATMENT of a Science over the fragmentary style so generally followed. BY A LONG WAY THE BEST of the small Manuals for Students."—*Analyst.*

LABORATORY HANDBOOKS BY A. HUMBOLDT SEXTON,
Professor of Metallurgy in the Glasgow and West of Scotland Technical College.

Sexton's (Prof.) Outlines of Quantitative Analysis.
FOR THE USE OF STUDENTS.

With Illustrations. FOURTH EDITION. Crown 8vo, Cloth, 3s.

"A COMPACT LABORATORY GUIDE for beginners was wanted, and the want has been WELL SUPPLIED. . . . A good and useful book."—*Lancet.*

Sexton's (Prof.) Outlines of Qualitative Analysis.
FOR THE USE OF STUDENTS.

With Illustrations. THIRD EDITION. Crown 8vo, Cloth, 3s. 6d.

"The work of a thoroughly practical chemist."—*British Medical Journal.*
"Compiled with great care, and will supply a want."—*Journal of Education.*

Sexton's (Prof.) Elementary Metallurgy:
Including the Author's Practical Laboratory Course. With many Illustrations. [See p. 66.

SECOND EDITION, Revised. Crown 8vo. Cloth, 6s.

"Just the kind of work for students commencing the study of metallurgy."—*Practical Engineer.*

CHEMISTRY FOR ENGINEERS AND MANUFACTURERS.

A PRACTICAL TEXT-BOOK.

BY

BERTRAM BLOUNT, AND **A. G. BLOXAM,**

F.I.C., F.C.S., Assoc.Inst.C.E.,
Consulting Chemist to the Crown Agents for the Colonies.

F.I.C., F.C.S.,
Consulting Chemist, Head of the Chemistry Department, Goldsmiths' Inst., New Cross.

With Illustrations. In Two Vols., Large 8vo. Sold Separately.

"The authors have SUCCEEDED beyond all expectations, and have produced a work which should give FRESH POWER to the Engineer and Manufacturer."—*The Times.*

VOLUME I. Price 10s. 6d.

CHEMISTRY OF ENGINEERING, BUILDING, AND METALLURGY.

General Contents.—**INTRODUCTION—Chemistry of the Chief Materials of Construction—Sources of Energy—Chemistry of Steam-raising—Chemistry of Lubrication and Lubricants—Metallurgical Processes used in the Winning and Manufacture of Metals.**

"PRACTICAL THROUGHOUT . . an ADMIRABLE TEXT-BOOK, useful not only to Students, but to ENGINEERS and MANAGERS OF WORKS in PREVENTING WASTE and IMPROVING PROCESSES."—*Scotsman.*

"EMINENTLY PRACTICAL."—*Glasgow Herald.*

"A book worthy of HIGH RANK . . . its merit is great . . . treatment of the subject of GASEOUS FUEL particularly good. . . . WATER GAS and the production clearly worked out. . . . Altogether a most creditable production. WE WARMLY RECOMMEND IT, and look forward with keen interest to the appearance of Vol. II."—*Journal of Gas Lighting.*

VOLUME II. Price 16s.

THE CHEMISTRY OF MANUFACTURING PROCESSES.

General Contents.—**Sulphuric Acid Manufacture—Manufacture of Alkali, &c.—Destructive Distillation—Artificial Manure Manufacture—Petroleum—Lime and Cement—Clay Industries and Glass—Sugar and Starch—Brewing and Distilling—Oils, Resins, and Varnishes—Soap and Candles—Textiles and Bleaching — Colouring Matters, Dyeing, and Printing — Paper and Pasteboard—Pigments and Paints—Leather, Glue, and Size—Explosives and Matches—Minor Chemical Manufactures.**

"Certainly a GOOD and USEFUL BOOK, constituting a PRACTICAL GUIDE for students by affording a clear conception of the numerous processes as a whole."—*Chemical Trade Journal.*

"We CONFIDENTLY RECOMMEND this volume as a PRACTICAL, and not overloaded, TEXT-BOOK, of GREAT VALUE to students."—*The Builder.*

WORKS BY A. WYNTER BLYTH, M.R.C.S., F.C.S.,

Barrister-at-Law, Public Analyst for the County of Devon, and Medical Officer of Health for St. Marylebone.

FOODS:

THEIR COMPOSITION AND ANALYSIS.

In Demy 8vo, with Elaborate Tables, Diagrams, and Plates. Handsome Cloth. FOURTH EDITION. Price 21s.

GENERAL CONTENTS.

History of Adulteration—Legislation, Past and Present—Apparatus useful to the Food-Analyst—"Ash"—Sugar—Confectionery—Honey-Treacle—Jams and Preserved Fruits—Starches—Wheaten-Flour—Bread—Oats—Barley—Rye—Rice—Maize—Millet—Potato—Peas—Chinese Peas—Lentils—Beans—MILK—Cream—Butter—Oleo-Margarine—Butterine—Cheese—Lard—Tea—Coffee—Cocoa and Chocolate—Alcohol—Brandy—Rum—Whisky—Gin—Arrack—Liqueurs—Absinthe—Principles of Fermentation—Yeast—Beer—Wine—Vinegar—Lemon and Lime Juice—Mustard—Pepper—Sweet and Bitter Almond—Annatto—Olive Oil—WATER—Standard Solutions and Reagents. *Appendix:* Text of English and American Adulteration Acts.

PRESS NOTICES OF THE FOURTH EDITION.

"Simply INDISPENSABLE in the Analyst's laboratory."—*The Lancet.*

"THE STANDARD WORK on the subject. . . . Every chapter and every page gives abundant proof of the strict revision to which the work has been subjected. . . . The section on MILK is, we believe, the most exhaustive study of the subject extant. . . . An INDISPENSABLE MANUAL for Analysts and Medical Officers of Health."—*Public Health.*

"A new edition of Mr. Wynter Blyth's Standard work, ENRICHED WITH ALL THE RECENT DISCOVERIES AND IMPROVEMENTS, will be accepted as a boon."—*Chemical News.*

POISONS:

THEIR EFFECTS AND DETECTION.

THIRD EDITION. In Large 8vo, Cloth, with Tables and Illustrations. Price 21s.

GENERAL CONTENTS.

I.—Historical Introduction. II.—Classification—Statistics—Connection between Toxic Action and Chemical Composition—Life Tests—General Method of Procedure—The Spectroscope—Examination of Blood and Blood Stains. III.—Poisonous Gases. IV.—Acids and Alkalies. V.—More or less Volatile Poisonous Substances. VI.—Alkaloids and Poisonous Vegetable Principles. VII.—Poisons derived from Living or Dead Animal Substances. VIII.—The Oxalic Acid Group. IX.—Inorganic Poisons. *Appendix:* Treatment, by Antidotes or otherwise, of Cases of Poisoning.

"Undoubtedly THE MOST COMPLETE WORK on Toxicology in our language."—*The Analyst (on the Third Edition).*

"As a PRACTICAL GUIDE, we know NO BETTER work."—*The Lancet (on the Third Edition).*

***** In the THIRD EDITION, Enlarged and partly Re-written, NEW ANALYTICAL METHODS have been introduced, and the CADAVERIC ALKALOIDS, or PTOMAINES, bodies playing so great a part in Food-poisoning and in the Manifestations of Disease, have received special attention.**

GRIFFIN'S TECHNOLOGICAL WORKS.

Large 8vo Volumes. Handsome Cloth.

With Numerous Tables, and 22 Illustrations. 16s.

DAIRY CHEMISTRY

FOR DAIRY MANAGERS, CHEMISTS, AND ANALYSTS

A Practical Handbook for Dairy Chemists and others having Control of Dairies.

By H. DROOP RICHMOND, F.C.S.,

CHEMIST TO THE AYLESBURY DAIRY COMPANY.

Contents.—I. Introductory.—The Constituents of Milk. II. The Analysis of Milk. III. Normal Milk: its Adulterations and Alterations, and their Detection. IV. The Chemical Control of the Dairy. V, Biological and Sanitary Matters. VI. Butter. VII. Other Milk Products. VIII. The Milk of Mammals other than the Cow.—Appendices.—Tables.—Index.

". . . In our opinion the book is the BEST CONTRIBUTION ON THE SUBJECT THAT HAS YET APPEARED in the English language."—*Lancet.*

"The author has succeeded in putting before the reader a complete book on Dairy Chemistry. It forms a COMPLETE *résumé* OF THEORETICAL AND PRACTICAL KNOWLEDGE, written in easy, intelligible language."—*The Analyst.*

MUNRO (J. M. H., D.Sc., Professor of Chemistry, Downton College of Agriculture):

AGRICULTURAL CHEMISTRY AND ANALYSIS: A PRACTICAL HAND-BOOK for the Use of Agricultural Students. (*Griffin's Technological Manuals.*) *In Preparation.*

CASTELL-EVANS (Prof. J., F.I.C., F.C.S., Finsbury Technical College):

TABLES AND DATA for the use of ANALYSTS, CHEMICAL MANUFACTURERS, and SCIENTIFIC CHEMISTS. In Large 8vo. Strongly Bound. [*At Press.*

*** This important Work will comprehend as far as possible ALL RULES AND TABLES required by the Analyst, Brewer, Distiller, Acid- and Alkali-Manufacturer, &c., &c.; and also the principal data in THERMO-CHEMISTRY, ELECTRO-CHEMISTRY, and the various branches of CHEMICAL PHYSICS which are constantly required by the Student and Worker in Original Research.

Every possible care has been taken to ensure perfect accuracy, and to include the results of the most recent investigations.

In Large 8vo. Handsome Cloth. With numerous Illustrations.

TECHNICAL MYCOLOGY:

THE UTILIZATION OF MICRO-ORGANISMS IN THE ARTS AND MANUFACTURES.

A Practical Handbook on Fermentation and Fermentative Processes for the Use of Brewers and Distillers, Analysts, Technical and Agricultural Chemists, and all interested in the Industries dependent on Fermentation.

BY DR. FRANZ LAFAR,

Professor of Fermentation-Physiology and Bacteriology in the Technical High School, Vienna.

With an Introduction by DR. EMIL CHR. HANSEN, **Principal of the Carlsberg Laboratory, Copenhagen.**

TRANSLATED BY CHARLES T. C. SALTER.

In Two Volumes, sold Separately.

Vol. I. now Ready. Complete in Itself. Price 15s.

"The first work of the kind which can lay claim to completeness in the treatment of a fascinating subject. The plan is admirable, the classification simple, the style is good, and the tendency of the whole volume is to convey sure information to the reader."—*Lancet.*

"We cannot sufficiently praise Dr. Lafar's work nor that of his admirable translator. No brewer with a love for his calling can allow such a book to be absent from his library."—*Brewer's Journal, New York.*

"We can most cordially recommend Dr. Lafar's volume to the Members of our profession . . . This treatise will supply a want felt in many industries. . . . No one will fail to observe how well Mr. Salter has done his work. The publishers have fully maintained their reputation as regards printing, binding, and excellence of paper."—*Chemical News.*

⁂ The publishers trust that before long they will be able to present English readers with the second volume of the above work, arrangements having been concluded whereby, upon its appearance in Germany the English translation will be at once put in hand.

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

In Large 8vo. Handsome Cloth. Price 21s.

THE PRINCIPLES AND PRACTICE OF
BREWING.

FOR THE USE OF STUDENTS AND PRACTICAL MEN.

BY

WALTER J. SYKES, M.D., D.P.H., F.I.C.,

EDITOR OF "THE ANALYST."

With Plate and Illustrations.

GENERAL CONTENTS.

I. **Physical Principles involved in Brewing Operations:** Heat: The Thermometer—Specific Heat—Latent Heat—Evaporation—Density and Specific Gravity—Hydrometers. **Chemistry, with special reference to the materials used in Brewing.**
II. **The Microscope:** General Description of the—Microscopical Manipulation—Examination of Yeast—Hanging-Drop Method—Examination of Bacteria—Microscopical Preparations—Bacteriological Methods—Bacteriological Examination of Water—Hansen's Method—Wichmann's Method—Bacteriological Examination of Air. **Vegetable Biology:** The Living Cell—Osmosis—The Yeasts—The Mycoderms—The Torulæ, &c.—The Bacteria—Fermentation and Putrefaction—Bacterium termo—Butyric Acid Bacteria, &c.—The Mould Fungi—Mucor mucedo, &c.—Simple Multicellular Organisms—Penicillium glaucum, &c.—Mould Fungi Dangerous on Brewing Premises—The Higher Plants—Germination of Barley—Structure of Barleycorn. **Fermentation:** Ancient Views of Liebig on—The Physiological Theory—Doctrine of Spontaneous Evolution—Sterilisation of Organic Fluids—Competition amongst Micro-Organisms—Distribution of Atmospheric Germs—Hansen's Investigations on the Air of Breweries—Pasteur's Experiments and Theory—Other Theories of Fermentation—Investigations of Hansen—Pure Cultures from a Single Cell—Introduction of Pure Yeast Cultures into the Brewery—Hansen's and other Pure Yeast Cultivation Apparatus—Advantages of Hansen's Pure Single-Cell Yeast—Differences in the Action of the various Yeasts
III. **Water:** Occurrence and Composition of—Results of Analysis of—Hardness—Waters Suitable for the Production of Different Classes of Ale—Artificial Treatment of Waters—Kainit—Influence of Boiling—Organic Constitution—Effect of Filtration—Methods of Water Analysis—Microscopic Examination of Water Sediments. **Barley and Malting:** Barley—Choice of—Vitality—Age—Malting—Steepings—Steep-Water—Germination of Barley—Flooring—Sprinkling—Withering—Pneumatic Malting—Galland's System—Saladin's—Hemming's—Drying Kiln—Changes Effected in Drying—Storage—Chemical Examination of Barley—Malt Substitutes—Quality of Malt—Chemical Examination of Malt—Ready-formed Sugars—Maltol. **Brewery Plant:** Gravitation Brewery—Cold and Hot Liquor Backs—Malt Mill—Mash Tun, &c.—Coppers—Coolers—Refrigerators—Collecting and Fermenting Vessels—Burton Union System—Attemperators—Parachutes—Racking Squares—Vats and Casks. **Brewing:** Estimation of Quantities for the Brew—Amount of Liquor Required—Hardening Materials—Mashing—Use of Subsidiary Apparatus—Black Beers—Sparging—Boiling—Action of Hop-tannin Bodies—Cooling—Refrigerating—Collection of Wort—Extract Yielded—Fermentation—Addition of Yeast—Change of Yeast—Fermentation Temperatures—Dressing—Appearance of Heads—Cleansing System—Stone Square System—Settling and Racking—Dry Hopping—Secondary Fermentation—Priming—Antiseptics—Fining—Bottled Ales and Bottling. **Beer and its Diseases:** Flavour and Aroma—Condition—Palate Fulness—Head—Brightness—Turbidity—Ropiness—Bibliography—**Appendices:** Solution Weight and Solution Factor—Specific Rotatory Power—The Law of Definite Relation—Alcoholic Fermentation without Yeast-Cells—Fermentation in a Vacuum—**Index.**

"A volume of Brewing Science, which has long been awaited. . . . We consider it one of THE MOST COMPLETE in CONTENTS and NOVEL IN ARRANGEMENT that has yet been published. . . . Will command a large sale."—*The Brewers' Journal.*

"The appearance of a work such as this serves to remind us of the ENORMOUSLY RAPID ADVANCES made in our knowledge of the Scientific Principles underlying the Brewing Processes. . . . Dr. Sykes' work will undoubtedly be of the GREATEST ASSISTANCE, not merely to Brewers, but to all Chemists and Biologists interested in the problems which the Fermentation industries present."—*The Analyst.*

"The publication of DR. SYKES' MASTERLY TREATISE on the art of Brewing is quite an event in the Brewing World. . . . Deserves our warmest praise. . . . A better guide than Dr. Sykes could hardly be found."—*County Brewers' Gazette.*

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

With Tables, Illustrations in the Text, and 37 Lithographic Plates. Medium 8vo. Handsome Cloth. 30s.

SEWAGE DISPOSAL WORKS:

A Guide to the Construction of Works for the Prevention of the Pollution by Sewage of Rivers and Estuaries.

BY W. SANTO CRIMP, M.INST.C.E., F.G.S.,
Late Assistant-Engineer, London County Council.

SECOND EDITION, REVISED AND ENLARGED.

PART I.—INTRODUCTORY. PART II.—SEWAGE DISPOSAL WORKS IN OPERATION—THEIR CONSTRUCTION, MAINTENANCE, AND COST.

Illustrated by Plates showing the General Plan and Arrangement adopted in each District.

⁂ From the fact of the Author's having, for some years, had charge of the Main Drainage Works of the Northern Section of the Metropolis, the chapter on LONDON will be found to contain many important details which would not otherwise have been available.

"All persons interested in Sanitary Science owe a debt of gratitude to Mr. Crimp. . . . His work will be especially useful to SANITARY AUTHORITIES and their advisers . . . EMINENTLY PRACTICAL AND USEFUL . . . gives plans and descriptions of MANY OF THE MOST IMPORTANT SEWAGE WORKS of England . . . with very valuable information as to the COST of construction and working of each. . . . The carefully-prepared drawings permit of an easy comparison between the different systems."—*Lancet.*

"Probably the MOST COMPLETE AND BEST TREATISE on the subject which has appeared in our language Will prove of the greatest use to all who have the problem of Sewage Disposal to face."—*Edinburgh Medical Journal.*

In Crown 8vo, Extra. With Illustrations. 8s. 6d.

CALCAREOUS CEMENTS:

THEIR NATURE, PREPARATION, AND USES.

With some Remarks upon Cement Testing.

BY GILBERT R. REDGRAVE, ASSOC. INST. C.E.,
Assistant Secretary for Technology, Board of Education, South Kensington.

GENERAL CONTENTS.—Introduction—Historical Review of the Cement Industry—The Early Days of Portland Cement—Composition of Portland Cement—PROCESSES OF MANUFACTURE—The Washmill and the Backs—Flue and Chamber Drying Processes—Calcination of the Cement Mixture—Grinding of the Cement—Composition of Mortar and Concrete—CEMENT TESTING — CHEMICAL ANALYSIS of Portland Cement, Lime, and Raw Materials — Employment of Slags for Cement Making — Scott's Cement, Selenitic Cement, and Cements produced from Sewage Sludge and the Refuse from Alkali Works — Plaster Cements — Specifications for Portland Cement—Appendices (Gases Evolved from Cement Works, Effects of Sea-water on Cement, Cost of Cement Manufacture, &c., &c.)

"A work calculated to be of GREAT and EXTENDED UTILITY."—*Chemical News.*

"INVALUABLE to the Student, Architect, and Engineer."—*Building News.*

"A work of the GREATEST INTEREST and USEFULNESS, which appears at a very critical period of the Cement Trade."—*Brit. Trade Journal.*

"Will be useful to ALL interested in the MANUFACTURE, USE, and TESTING of Cements."—*Engineer.*

Now Ready. *Large 8vo. Handsome Cloth. Beautifully Illustrated.* 21s.

Road Making and Maintenance:

A PRACTICAL TREATISE FOR ENGINEERS, SURVEYORS, AND OTHERS.

WITH AN HISTORICAL SKETCH OF ANCIENT AND MODERN PRACTICE.

BY THOS. AITKEN, Assoc.M.Inst.C.E.,

Member of the Association of Municipal and County Engineers; Member of the Sanitary Inst.; Surveyor to the County Council of Fife, Cupar Division.

WITH NUMEROUS PLATES, DIAGRAMS, AND ILLUSTRATIONS.

Contents.—Historical Sketch.—Resistance of Traction.—Laying out New Roads. — Earthworks, Drainage, and Retaining Walls. — Road Materials, or Metal.—Quarrying.—Stone Breaking and Haulage.—Road-Rolling and Scarifying.—The Construction of New, and the Maintenance of existing Roads.—Carriage Ways and Foot Ways.

"The Literary style is excellent. . . . A comprehensive and excellent Modern Book, an up-to-date work. . . . Should be on the reference shelf of every Municipal and County Engineer or Surveyor in the United Kingdom, and of every Colonial Engineer."—*The Surveyor.*

Second Edition, Revised and Enlarged, with New Section on Acetylene. Fully Illustrated. 10s. 6d.

THE CHEMISTRY OF

GAS MANUFACTURE.

A Hand-Book on the Production, Purification, and Testing of Illuminatin Gas, and the Assay of the Bye-Products of Gas Manufacture. For the Use of Students.

BY

W. J. ATKINSON BUTTERFIELD, M.A., F.I.C., F.C.S.,

Formerly Head Chemist, Gas Works, Beckton, London, E.

General Contents.—I. Raw Materials for Gas Manufacture.—II. Coal Gas.—III. Carburetted Water Gas.—IV. Oil Gas.—V. Enriching by Light Oils.—VI.—Final Details of Manufacture.—VII. Gas Analysis.—VIII. Photometry.—IX. Applications of Gas.—X. Bye-Products.—XI. Acetylene.—Index.

"The best work of its kind which we have ever had the pleasure of reviewing."—*Journal of Gas Lighting.*

"Amongst works not written in German, we recommend before all others, Butterfield's Chemistry of Gas Manufacture."—*Chemiker Zeitung.*

SEVENTH EDITION, Revised. Price 6s.

PRACTICAL SANITATION:

A HAND-BOOK FOR SANITARY INSPECTORS AND OTHERS INTERESTED IN SANITATION.

By GEORGE REID, M.D., D.P.H.,

Fellow, Mem. Council, and Examiner, Sanitary Institute of Great Britain, and Medical Officer to the Staffordshire County Council.

With an Appendix on Sanitary Law.

By HERBERT MANLEY, M.A., M.B., D.P.H.,

Medical Officer of Health for the County Borough of West Bromwich.

GENERAL CONTENTS.—Introduction—Water Supply: Drinking Water, Pollution of Water—Ventilation and Warming—Principles of Sewage Removal—Details of Drainage; Refuse Removal and Disposal—Sanitary and Insanitary Work and Appliances—Details of Plumbers' Work—House Construction—Infection and Disinfection—Food, Inspection of; Characteristics of Good Meat; Meat, Milk, Fish, &c., unfit for Human Food—Appendix: Sanitary Law; Model Bye-Laws, &c.

"Dr. Reid's very useful Manual . . . ABOUNDS IN PRACTICAL DETAIL." —*British Medical Journal.*

"A VERY USEFUL HANDBOOK, with a very useful Appendix. We recommend it not only to SANITARY INSPECTORS, but to HOUSEHOLDERS and ALL interested in Sanitary matters."—*Sanitary Record.*

NOW READY. Crown 8vo, Handsome Cloth. Fully Illustrated. 10s. 6d.

FLESH FOODS:

With Methods for their Chemical, Microscopical, and Bacteriological Examination.

A Practical Handbook for Medical Men, Analysts, Inspectors and others.

BY C. AINSWORTH MITCHELL, B.A.(OXON),

Fellow of the Institute of Chemistry; Member of Council, Society of Public Analysts.

With Numerous Tables, Illustrations, and a Coloured Plate.

CONTENTS.—Structure and Chemical Composition of Muscular Fibre.—of Connective Tissue, and Blood.—The Flesh of Different Animals.—The Examination of Flesh.—Methods of Examining Animal Fat.—The Preservation of Flesh.—Composition and Analysis of Sausages.—Proteids of Flesh.—Meat Extracts and Flesh Peptones.—The Cooking of Flesh.—Poisonous Flesh.—The Animal Parasites of Flesh.—The Bacteriological Examination of Flesh.—The Extraction and Separation of Ptomaines.—INDEX.

⁂ This work is a complete compendium of the CHEMISTRY OF ANIMAL TISSUES. It contains directions for the detection of morbid conditions, putrefactive changes, and poisonous or injurious constituents, together with an account of their causes and effects.—*Publisher's Note.*

"A compilation which will be most useful for the class for whom it is intended."—*Athenæum.*

"A book which NO ONE whose duties involve considerations of food supply CAN AFFORD TO BE WITHOUT."—*Municipal Journal.*

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

COMPANION VOLUME TO REID'S SANITATION.

In Crown 8vo. Handsome Cloth. Profusely Illustrated.

Sanitary Engineering:

A Practical Manual of Town Drainage and Sewage and Refuse Disposal.

For Sanitary Authorities, Engineers, Inspectors, Architects, Contractors, and Students.

BY

FRANCIS WOOD, A.M.INST.C.E., F.G.S.,

Borough Engineer, Bacup, Lancs.

GENERAL CONTENTS.

Introduction.—Hydraulics.—Velocity of Water in Pipes.—Earth Pressures and Retaining Walls.—Powers.—House Drainage.—Land Drainage.—Sewers.—Separate System.—Sewage Pumping.—Sewer Ventilation.—Drainage Areas.—Sewers, Manholes, &c.—Trade Refuse.—Sewage Disposal Works.—BACTERIOLYSIS.—Sludge Disposal.—Construction and Cleansing of Sewers.—Refuse Disposal.—Chimneys and Foundations.

[AT PRESS.

READY SHORTLY. IMPORTANT NEW WORK.

Large 8vo. Handsome Cloth.

WITH NUMEROUS PLATES, DIAGRAMS, AND ILLUSTRATIONS.

Central Electrical Stations:

THEIR DESIGN, ORGANISATION, AND MANAGEMENT.

BY

CHAS. H. WORDINGHAM,

A.K.C., M.Inst.C.E., M.Inst.M.E.,

Late Member of Council, Institution of Electrical Engineers; Engineer to the City and Electric Tramways, Manchester.

For Contents, see p. 43.

Painters' Colours, Oils, & Varnishes:

A PRACTICAL MANUAL.

By GEORGE H. HURST, F.C.S.,

Member of the Society of Chemical Industry; Lecturer on the Technology of Painters' Colours, Oils, and Varnishes, the Municipal Technical School, Manchester.

Second Edition, Revised and Enlarged. With Illustrations. 12s. 6d.

General Contents.—Introductory—The Composition, Manufacture, Assay, and Analysis of Pigments, White, Red, Yellow and Orange, Green, Blue, Brown, and Black—Lakes—Colour and Paint Machinery—Paint Vehicles (Oils, Turpentine, &c., &c.)—Driers—Varnishes.

"This useful book will prove most valuable."—*Chemical News.*

"A *practical* manual in every respect . . . exceedingly instructive. The section on Varnishes is the most reasonable we have met with."—*Chemist and Druggist.*

"Very valuable information is given."—*Plumber and Decorator.*

"A thoroughly practical book, . . . the only English work that satisfactorily treats of the manufacture of oils, colours, and pigments."—*Chemical Trades' Journal.*

⁂ For Mr. Hurst's Garment Dyeing and Cleaning, see p. 84.

WORKS BY DR. ALDER WRIGHT, F.R.S.

Fixed Oils, Fats, Butters, and Waxes:

THEIR PREPARATION AND PROPERTIES,

AND THE

MANUFACTURE THEREFROM OF CANDLES, SOAPS, AND OTHER PRODUCTS.

BY

C. R. ALDER WRIGHT, D.Sc., F.R.S.,

Late Lecturer on Chemistry, St. Mary's Hospital Medical School; Examiner in "Soap" to the City and Guilds of London Institute.

In Large 8vo. Handsome Cloth. With 144 Illustrations. 28s.

"Dr. Wright's work will be found absolutely indispensable by every Chemist. Teems with information valuable alike to the Analyst and the Technical Chemist."—*The Analyst.*

"Will rank as the Standard English Authority on Oils and Fats for many years to come."—*Industries and Iron.*

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

Painting and Decorating:

A Complete Practical Manual for House Painters and Decorators.

BY

WALTER JOHN PEARCE,

LECTURER AT THE MANCHESTER TECHNICAL SCHOOL FOR HOUSE-PAINTING AND DECORATING.

In Crown 8vo. extra. With Numerous Illustrations and Plates (some in Colours), including Original Designs. 12s. 6d.

GENERAL CONTENTS.

Introduction—Workshop and Stores—Plant and Appliances—Brushes and Tools—Materials: Pigments, Driers, Painters' Oils—Wall Hangings—Paper Hanging—Colour Mixing—Distempering—Plain Painting—Staining—Varnish and Varnishing—Imitative Painting—Graining—Marbling—Gilding—Sign-Writing and Lettering—Decoration: General Principles—Decoration in Distemper—Painted Decoration—Relievo Decoration—Colour—Measuring and Estimating—Coach-Painting—Ship-Painting.

"A THOROUGHLY USEFUL BOOK . . . gives GOOD, SOUND, PRACTICAL INFORMATION in a CLEAR and CONCISE FORM. . . . Can be confidently recommended alike to Student and Workman, as well as to those carrying on business as House Painters and Decorators."—*Plumber and Decorator.*

"A THOROUGHLY GOOD AND RELIABLE TEXT-BOOK. . . . So FULL and COMPLETE that it would be difficult to imagine how anything further could be added about the Painter's craft."—*Builders' Journal.*

SECOND EDITION. Crown 8vo, Handsome Cloth. 21*s.*

Including all the Newer Developments in Photographic Methods, together with Special Articles on Radiography (the X-Rays), Colour Photography, and many New Plates.

PHOTOGRAPHY:

ITS HISTORY, PROCESSES, APPARATUS, AND MATERIALS.

A PRACTICAL MANUAL.

Comprising Working Details of all the More Important Methods.

BY A. BROTHERS, F.R.A.S.

WITH NUMEROUS FULL-PAGE PLATES BY MANY OF THE PROCESSES DESCRIBED, AND ILLUSTRATIONS IN THE TEXT.

"A standard work on Photography brought quite up-to-date."—*Photography.*

"A highly informative book. . . . We can cordially recommend the volume as a worthy addition to any library."—*British Journal of Photography.*

"Pre-eminently a sound practical treatise on Photography written by a practical worker of life-long experience."—*Leeds Mercury.*

"The illustrations are of great beauty."—*Scotsman.*

---

§10. THE TEXTILE INDUSTRIES.

"The MOST VALUABLE and USEFUL WORK on Dyeing that has yet appeared in the English language . . . likely to be THE STANDARD WORK OF REFERENCE for years to come."—*Textile Mercury.*

In Two Large 8vo Volumes, 920 pp., with a SUPPLEMENTARY Volume, containing Specimens of Dyed Fabrics. 45s.

A MANUAL OF DYEING:

FOR THE USE OF PRACTICAL DYERS, MANUFACTURERS, STUDENTS, AND ALL INTERESTED IN THE ART OF DYEING.

BY

E. KNECHT, Ph.D., F.I.C.,	**CHR. RAWSON, F.I.C., F.C.S.,**
Head of the Chemistry and Dyeing Department of the Technical School, Manchester; Editor of "The Journal o the Society of Dyers and Colourists;"	Late Head of the Chemistry and Dyeing Department of the Technical College, Bradford; Member of Council of the Society of Dyers and Colourists;

And RICHARD LOEWENTHAL, Ph.D.

GENERAL CONTENTS.—Chemical Technology of the Textile Fabrics—Water—Washing and Bleaching—Acids, Alkalies, Mordants—Natural Colouring Matters—Artificial Organic Colouring Matters—Mineral Colours—Machinery used in Dyeing—Tinctorial Properties of Colouring Matters—Analysis and Valuation of Materials used in Dyeing, &c., &c.

"This MOST VALUABLE WORK . . will be widely appreciated."—*Chemical News.*

"This authoritative and exhaustive work . . . the MOST COMPLETE we have yet seen on the subject."—*Textile Manufacturer.*

"The MOST EXHAUSTIVE and COMPLETE WORK on the subject extant."—*Textile Recorder.*

"The distinguished authors have placed in the hands of those daily engaged in the dye-house or laboratory a work of EXTREME VALUE and UNDOUBTED UTILITY . . . appeals quickly to the technologist, colour chemist, dyer, and more particularly to the rising dyer of the present generation. A book which it is refreshing to meet with."—*American Textile Record.*

Large 8vo. Handsome Cloth.

A DICTIONARY OF DYESTUFFS.

A Compendium of Dyes, Mordants, and Other Substances Employed in Dyeing, Calico-Printing, and Bleaching.

BY C. RAWSON, F.I.C., F.C.S., W. M. GARDNER, F.C.S., AND W. F. LAYCOCK, Ph.D., F.C.S.

With Formulæ, Properties, Applications, &c.

[*At Press.*

Companion-Volume to Knecht and Rawson's "Dyeing."

TEXTILE PRINTING:

A PRACTICAL MANUAL.

Including the Processes Used in the Printing of
COTTON, WOOLLEN, SILK, and HALF-SILK FABRICS.

BY

C. F. SEYMOUR ROTHWELL, F.C.S.,

Mem. Soc. of Chemical Industries; late Lecturer at the Municipal Technical School, Manchester.

In Large 8vo, with Illustrations and Printed Patterns. Price 21s.

GENERAL CONTENTS.

Introduction.
The Machinery Used in Textile Printing.
Thickeners and Mordants.
The Printing of Cotton Goods.
The Steam Style.
Colours Produced Directly on the Fibre.
Dyed Styles.
Padding Style.
Resist and Discharge Styles.
The Printing of Compound Colourings, &c.
The Printing of Woollen Goods.
The Printing of Silk Goods.
Practical Recipes for Printing.
Appendix.
Useful Tables.
Patterns.

"By far the best and most practical book on textile printing which has yet been brought out, and will long remain the standard work on the subject. It is essentially practical in character."—*Textile Mercury.*

"The most practical manual of textile printing which has yet appeared. We have no hesitation in recommending it."—*The Textile Manufacturer.*

"Undoubtedly Mr. Rothwell's book is the best which has appeared on textile printing, and worthily forms a Companion-Volume to 'A Manual on Dyeing.'"—*The Dyer and Calico Printer.*

Large 8vo. Handsome Cloth.

A Dictionary of Textile Fibres.

By WILLIAM J. HANNAN,

Lecturer on Botany at the Ashton Municipal Technical School, Lecturer on Cotton Spinning at the Chorley Science and Art School, &c.

With Numerous Illustrations reproduced from Photographs, &c.

[*In Preparation.*

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.

Large 8vo. Handsome Cloth. 12s. 6d.

BLEACHING & CALICO-PRINTING.

A Short Manual for Students and Practical Men.

BY GEORGE DUERR,

Director of the Bleaching, Dyeing, and Printing Department at the Accrington and Bacup Technical Schools; Chemist and Colourist at the Irwell Print Works.

ASSISTED BY WILLIAM TURNBULL

(of Turnbull & Stockdale, Limited).

With Illustrations and upwards of One Hundred Dyed and Printed Patterns designed specially to show various Stages of the Processes described.

GENERAL CONTENTS.—COTTON, Composition of; BLEACHING, New Processes; PRINTING, Hand-Block; Flat-Press Work; Machine Printing—MORDANTS—STYLES OF CALICO-PRINTING: The Dyed or Madder Style, Resist Padded Style, Discharge and Extract Style, Chromed or Raised Colours, Insoluble Colours, &c. — Thickeners — Natural Organic Colouring Matters —Tannin Matters — Oils, Soaps, Solvents — Organic Acids — Salts — Mineral Colours—Coal Tar Colours—Dyeing—Water, Softening of—Theory of Colours —Weights and Measures, &c.

"When a READY WAY out of a difficulty is wanted, it is IN BOOKS LIKE THIS that it is found."—*Textile Recorder.*

"Mr. DUERR'S WORK will be found MOST USEFUL. . . . The information given is of GREAT VALUE. . . . The Recipes are THOROUGHLY PRACTICAL."—*Textile Manufacturer.*

GARMENT DYEING AND CLEANING.

A Practical Book for Practical Men.

BY GEORGE H. HURST, F.C.S.,

Member of the Society of Chemical Industry.

With Numerous Illustrations. 4s. 6d.

GENERAL CONTENTS.—Technology of the Textile Fibres—Garment Cleaning —Dyeing of Textile Fabrics—Bleaching—Finishing of Dyed and Cleaned Fabrics—Scouring and Dyeing of Skin Rugs and Mats—Cleaning and Dyeing of Feathers—Glove Cleaning and Dyeing—Straw Bleaching and Dyeing—Glossary of Drugs and Chemicals—Useful Tables.

"An UP-TO-DATE hand book has long been wanted, and Mr. Hurst has done nothing more complete than this. An important work, the more so that several of the branches of the craft here treated upon are almost entirely without English Manuals for the guidance of workers. The price brings it within the reach of all."—*Dyer and Calico-Printer.*

"Mr. Hurst's WORK DECIDEDLY FILLS A WANT . . ought to be in the hands of EVERY GARMENT DYER and cleaner in the Kingdom"—*Textile Mercury.*

"BOYS COULD NOT HAVE A MORE ALLURING INTRODUCTION to scientific pursuits than these charming-looking volumes."—Letter to the Publishers from the Head-master of one of our great Public Schools.

OPEN-AIR STUDIES IN BOTANY:

SKETCHES OF BRITISH WILD FLOWERS IN THEIR HOMES.

BY

R. LLOYD PRAEGER, B.A., M.R.I.A.

Illustrated by Drawings from Nature by S. Rosamond Praeger, and Photographs by R. Welch.

Handsome Cloth, 7s. 6d. Gilt, for Presentation, 8s. 6d.

GENERAL CONTENTS.—A Daisy-Starred Pasture—Under the Hawthorns—By the River—Along the Shingle—A Fragrant Hedgerow—A Connemara Bog—Where the Samphire grows—A Flowery Meadow—Among the Corn (a Study in Weeds)—In the Home of the Alpines—A City Rubbish-Heap—Glossary.

"A FRESH AND STIMULATING book . . . should take a high place . . . The Illustrations are drawn with much skill."—*The Times.*

"BEAUTIFULLY ILLUSTRATED. . . . One of the MOST ACCURATE as well as INTERESTING books of the kind we have seen."—*Athenæum.*

"Redolent with the scent of woodland and meadow."—*The Standard.*

"A Series of STIMULATING and DELIGHTFUL Chapters on Field-Botany."—*The Scotsman.*

"A work as FRESH in many ways as the flowers themselves of which it treats. The RICH STORE of information which the book contains . . ."—*The Garden.*

OPEN-AIR STUDIES IN GEOLOGY:

An Introduction to Geology Out-of-doors.

BY

GRENVILLE A. J. COLE, F.G.S., M.R.I.A.,

Professor of Geology in the Royal College of Science for Ireland.

With 12 Full-Page Illustrations from Photographs. Cloth. 8s. 6d.

GENERAL CONTENTS.—The Materials of the Earth—A Mountain Hollow—Down the Valley—Along the Shore—Across the Plains—Dead Volcanoes—A Granite Highland—The Annals of the Earth—The Surrey Hills—The Folds of the Mountains.

"The FASCINATING 'OPEN-AIR STUDIES' of PROF. COLE give the subject a GLOW OF ANIMATION . . . cannot fail to arouse keen interest in geology."—*Geological Magazine.*

"EMINENTLY READABLE . . . every small detail in a scene touched with a sympathetic kindly pen that reminds one of the lingering brush of a Constable."—*Nature.*

"The work of Prof. Cole combines ELEGANCE of STYLE with SCIENTIFIC THOROUGHNESS."—*Petermann's Mittheilungen.*

"The book is worthy of its title; from cover to cover it is STRONG with bracing freshness of the mountain and the field, while its ACCURACY and THOROUGHNESS show that it is the work of an earnest and conscientious student. . . . Full of picturesque touches which are most welcome."—*Natural Science.*

"A CHARMING BOOK, beautifully illustrated."—*Athenæum.*

Eighteenth Annual Issue. Handsome cloth, 7s. 6d.

THE OFFICIAL YEAR-BOOK

OF THE

SCIENTIFIC AND LEARNED SOCIETIES OF GREAT BRITAIN AND IRELAND.

COMPILED FROM OFFICIAL SOURCES.

Comprising (together with other Official Information) LISTS of the PAPERS read during 1900 before all the LEADING SOCIETIES throughout the Kingdom engaged in the following Departments of Research:—

- § 1. Science Generally: *i.e.*, Societies occupying themselves with several Branches of Science, or with Science and Literature jointly.
- § 2. Mathematics and Physics.
- § 3. Chemistry and Photography.
- § 4. Geology, Geography, and Mineralogy.
- § 5. Biology, including Microscopy and Anthropology.
- § 6. Economic Science and Statistics.
- § 7. Mechanical Science, Engineering, and Architecture
- § 8. Naval and Military Science.
- § 9. Agriculture and Horticulture
- § 10. Law.
- § 11. Literature.
- § 12 Psychology.
- § 13. Archæology.
- § 14. MEDICINE.

"'The Year-Book of Societies' FILLS A VERY REAL WANT."—*Engineering.*

"INDISPENSABLE to any one who may wish to keep himself abreast of the scientific work of the day."—*Edinburgh Medical Journal.*

"The YEAR-BOOK OF SOCIETIES is a Record which ought to be of the greatest use for the progress of Science."—*Lord Playfair, F.R.S., K.C.B., M.P., Past-President of the British Association.*

"It goes almost without saying that a Handbook of this subject will be in time one of the most generally useful works for the library or the desk."—*The Times.*

"British Societies are now well represented in the 'Year-Book of the Scientific and Learned Societies of Great Britain and Ireland.'"—(Art. "Societies" in New Edition of "Encyclopædia Britannica," vol. xxii.)

Copies of the FIRST ISSUE, giving an Account of the History, Organization, and Conditions of Membership of the various Societies, and forming the groundwork of the Series, may still be had, price 7/6. *Also Copies of the Issues following.*

The YEAR-BOOK OF SOCIETIES forms a complete INDEX TO THE SCIENTIFIC WORK of the sessional year in the various Departments. It is used as a HANDBOOK in all our great SCIENTIFIC CENTRES, MUSEUMS, and LIBRARIES throughout the Kingdom, and has become an INDISPENSABLE BOOK OF REFERENCE to every one engaged in Scientific Work.

READY IN SEPTEMBER EACH YEAR.

LONDON: CHARLES GRIFFIN & CO., LIMITED, EXETER STREET, STRAND.